Douglass Houghton Workshop, Section 2, Thu 12/04/25

Worksheet What the World Needs Now

1. Natalie P. is on the ice, doing a scratch spin with her arms out. When she brings her arms in to make an angle of ϕ with her body, her angular velocity, in radians per second, will be

$$\omega = \frac{L}{B + rA\sin\phi}$$

where L, B, r, and A are constants. (L is Natalie's *angular momentum*, B is her body's *moment of inertia*, r is the length of her arms, and A is the moment of inertia of her arms.) So the velocity of her hands is

$$v = \frac{Lr\sin\phi}{A + B\sin\phi}$$

If $d\phi/dt = 10$ radians per second, how fast are her hands accelerating, in terms of the other constants?

Natalie is holding a Chai cookie in one of her hands, and eventually it's accelerating fast enough to fly off and land in a nearby bowling alley.

2. Natalie E. is practicing her bowling. For some reason, there is a cookie sitting 50 feet down the lane from the foul line. Natalie E. throws a series of balls, each with different velocities. t is the time after the ball passes the foul line. Fill in the table below:

Velocity (ft/sec)	Position of the ball at time T	Does it hit the cookie?
$90e^{-2t}$		
$96/(t+1)^3$		
$1/(t+1)$		

3. Jillian is escaping from the hospital with Katie's help. The plan is this: Jillian will jump out a window 400 feet above the ground, and Katie will catch her by driving a mattress under the window at just the right moment. Let t be the time since the jump.

   (a) Jillian accelerates at $32\,\mathrm{ft/s^2}$ as she falls. Find formulas for her velocity $v(t)$ and height $h(t)$ at time t. When will she reach the ground?

   (b) Katie is waiting 40 feet away behind some bushes. Find

   - The distance $L(t)$ between Jillian and Katie at time t,
   - $L'(t)$. (Can write this in terms of $v(t)$,h(t), and $L(t)$.)

   (c) When Katie receives a signal, she'll floor the accelerator to get the truck's maximum acceleration of $10\,\mathrm{ft}/s^2$. How long will it take for her to be halfway to the pickup point?

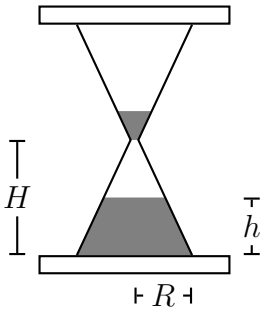
   (d) When she's halfway there, Katie will slam on the brakes and decelerate at the same rate. So double the previous answer to get the total time it will take Katie to be in position to catch Jillian. By when must she start the truck in order to be in time to catch Jillian?

(e) Katie's signal to proceed is an audio tone from Jillian. Katie is expecting it to be at 500 Hz, but due to an error in tuning, Jillian is transmitting at 490 Hz. However, since the speed of sound is 1125 ft/s, the Doppler effect will cause Katie to receive the signal at

$$490\frac{1125}{1125 + L'(t)}\,\text{Hz}.$$

So the question is, will Katie receive the correct signal before it's too late?

4. The lower chamber of an hourglass is shaped like a cone with height H inches and base radius R inches, as shown in the figure to the right, above. Sand falls into this cone. Write an expression for the volume of the sand in the lower chamber when the height of the sand there is h in (Hint: A cone with base radius r and height y has volume $V = \frac{1}{3}\pi r^2 y$, and it may be helpful to think of a difference between two conical vol umes.) Then, if $R = 0.9$ in, $H = 2.7$ in, and sand is falling into the lower chamber at $2\,\text{in}^3/\text{min}$, how fast is the height of the sand in the lower chamber changing when $h = 1$ in?



5. (This problem appeared on a Fall, 2008 Math 115 exam) Determine a and b for the function of the form $y = f(t) = at^2 + b/t$, with a local minimum at $(1, 12)$.

6. (From the Winter, 2007 Math 115 Final Exam) Suppose that f and g are continuous functions with

$$\int_0^2 f(x)\,dx = 5 \qquad \text{and} \qquad \int_0^2 g(x)\,dx = 13.$$

Compute the following. If the computation cannot be made because something is missing, explain clearly what is missing.

(a) $\displaystyle\int_4^6 f(x-4)\,dx$

(b) $\displaystyle\int_{-2}^0 2g(-t)\,dt$

(c) $\displaystyle\int_2^0 (f(y)+2)\,dy$

(d) $\displaystyle\int_2^2 g(x)\,dx$

(e) Suppose that f is an even function. Find the average value of f from -2 to 2.

7. Here is the graph of the *derivative* of the continuous function $M(x)$. Using the fact that $M(-4) = -2$, sketch the graph of $M(x)$. Give the coordinates of all critical points, inflection points, and endpoints.