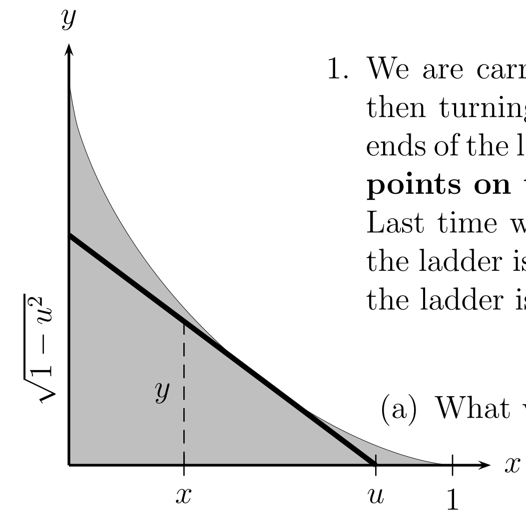
Douglass Houghton Workshop, Section 2, Tue 12/02/25

Worksheet Veni, Vidi, Vici



1. We are carrying a ladder of length 1 down a hallway, and then turning it to get around a corner, always keeping the ends of the ladder against the walls. The question is: **Which points on the floor does the ladder pass over?**
Last time we found that if $0 \le x \le u \le 1$ and the base of the ladder is at $(u, 0)$, then the distance from $(x, 0)$ north to the ladder is

$$y = \frac{u-x}{u}\sqrt{1-u^2}.$$

   (a) What value of u maximizes y? (Keep x fixed!)

   (b) So for a fixed x, what is the maximum value of y, as the ladder moves?

   (c) You have found a formula for the curve at the top of the region we want. Simplify until it's beautiful. (This is the best part, so don't stop until it's truly wondrous.)

2. (Fall, 2014) Addy decides to turn her passion for earrings into a business. After careful study, Addy has determined that she can produce up to 160 pairs of earrings in a year. She can sell the first 100 pairs of earrings to jewelry stores and any remaining earrings to wholesalers. The revenue in dollars from selling x pairs of earrings will be

$$R(x) = \begin{cases} 6x & \text{if } 0 \le x \le 100 \\ 4x + 200 & \text{if } 100 < x \le 160. \end{cases}$$

   (a) What is the price jewelry stores pay for each pair of earrings?

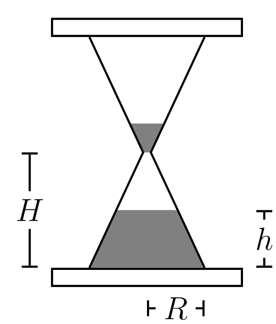
   (b) What is the price wholesalers pay?

   (c) It costs $C(x) = 20 + 3x + 24\sqrt{x}$ to produce x pairs of earrings. (Use that formula for the rest of the problem.) What is the fixed cost of Addy's operation?

   (d) At what production levels does marginal revenue equal marginal cost?

   (e) How many pairs of earrings should Addy produce to maximize her profit, and what is the maximum possible profit?

3. The lower chamber of an hourglass is shaped like a cone with height H inches and base radius R inches, as shown in the figure to the right, above. Sand falls into this cone. Write an expression for the volume of the sand in the lower chamber when the height of the sand there is h in (Hint: A cone with base radius r and height y has volume $V = \frac{1}{3}\pi r^2 y$, and it may be helpful to think of a difference between two conical vol umes.) Then, if $R = 0.9$ in, $H = 2.7$ in, and sand is falling into the lower chamber at $2\,\text{in}^3/\text{min}$, how fast is the height of the sand in the lower chamber changing when $h = 1$ in?



4. (This problem appeared on a Fall, 2008 Math 115 exam) Determine a and b for the function of the form $y = f(t) = at^2 + b/t$, with a local minimum at $(1, 12)$.

5. (From the Winter, 2007 Math 115 Final Exam) Suppose that f and g are continuous functions with

$$\int_0^2 f(x)\,dx = 5 \qquad \text{and} \qquad \int_0^2 g(x)\,dx = 13.$$

Compute the following. If the computation cannot be made because something is missing, explain clearly what is missing.

(a) $\int_4^6 f(x-4)\,dx$

(b) $\int_{-2}^0 2g(-t)\,dt$

(c) $\int_2^0 (f(y)+2)\,dy$

(d) $\int_2^2 g(x)\,dx$

(e) Suppose that f is an even function. Find the average value of f from -2 to 2.

6. Here is the graph of the *derivative* of the continuous function $M(x)$. Using the fact that $M(-4) = -2$, sketch the graph of $M(x)$. Give the coordinates of all critical points, inflection points, and endpoints.