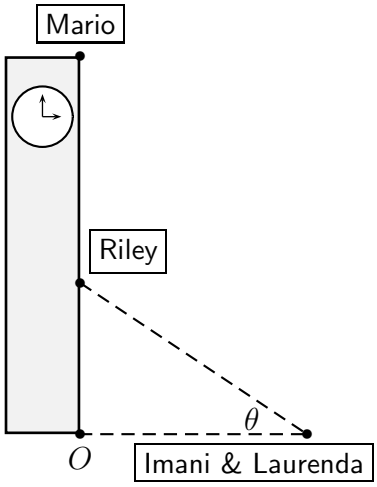
Douglass Houghton Workshop, Section 2, Tue 11/18/25

Worksheet To Infinity, and Beyond

1. (Adapted from a Winter, 2005 Math 115 exam) One day Mario notices that the door to the Burton Tower carillon has been left open. He can't resist the urge to climb to the top of the tower and barricade himself in. He then launches in to his solo from *Vive l'amour*, followed by the rest of the Men's Glee Club repertoire.
Riley, observing that there is a ledge protruding from a second-story window in the tower, pole vaults 9 feet in the air to grab the ledge and begins climbing the outside of the tower. Meanwhile on the ground, 30 feet from the tower, Imani is giving an impromptu Indian dance demonstration, accompanied by Laurenda on the violin. They look up at an angle θ to see Riley.



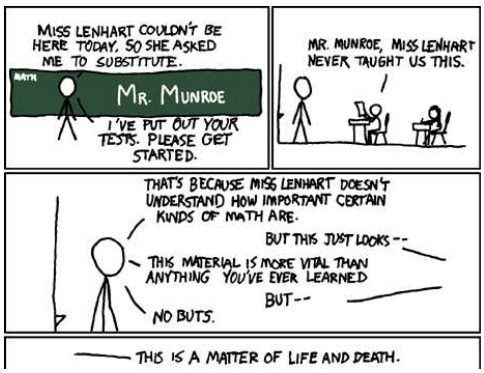
   (a) Find the rate of change of Riley's distance from the point O with respect to θ.

   (b) If the distance from point O to Mario is 200 ft and Riley climbs at a constant 8 ft/sec, what is the rate of change of θ with respect to time when Riley is halfway up?

   (c) When Riley is halfway up, Mario drops the end of a a rope down to help her. The end of the rope falls with a constant acceleration of 32 ft/sec^2. When does Riley catch it, and what is its speed when she does?

   (d) Imani & Laurenda watch the end of the rope as it drops, and also begin backing away from the tower at a rate of 5 ft/sec. How fast is the angle of their gaze changing when Riley catches the rope?

2.

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The velociraptor spots you 40 meters away and
attacks, accelerating at 4 m/s^2 up to its top
speed of 25 m/s.  When it spots you, you begin
to flee, quickly reaching your top speed of 6
m/s.  How far can you get before you're caught
and devoured?
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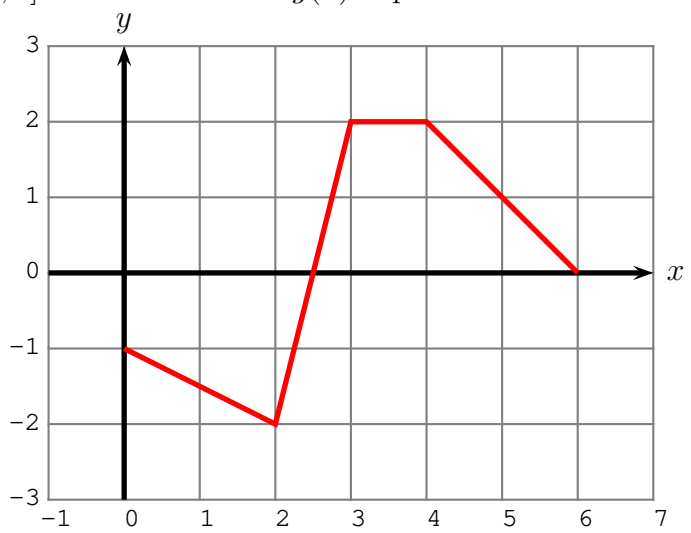
3. (Winter, 2024) An **even** function $g(x)$, which is defined for all real numbers, is graphed on the interval $[0, 6]$ below. Note that $g(x)$ is piecewise linear.



(a) The function $g(x)$ has a continuous antiderivative, $G(x)$, which satisfies $G(2) = -2$. Complete the following table of values for $G(x)$.

x	-6	-4	-2	0	2	4	6
$G(x)$					-2		

(b) Sketch a graph of $G(x)$ on the interval $[-6, 6]$. Make sure to clearly label the values at the points in your table above and also make it clear where $G(x)$ is increasing or decreasing, and where $G(x)$ is concave up, concave down, or linear.

4. (Winter, 2010) Suppose that the standard price of a round-trip plane ticket from Detroit to Paris, purchased t days after April 30, is $P(t)$ dollars. Assume that P is an invertible function (even though this is not always the case in real life). In the context of this problem, give a practical interpretation for each of the following:

(a) $P'(2) = 55$

(b) $\displaystyle\int_5^{10} P'(t)\,dt$

(c) $P^{-1}(690)$

(d) $\displaystyle\frac{1}{5}\int_5^{10} P(t)\,dt$

5. (Winter, 2012) Consider the family of functions

$$f(x) = ax - e^{bx},$$

where a and b are positive constants.

(a) Any function $f(x)$ in this family has only one critical point. In terms of a and b, what are the x- and y-coordinates of that critical point?

(b) Is the critical point a local maximum or a local minimum? Justify your answer with either the first-derivative test or the second-derivative test.

(c) For which values of a and b will $f(x)$ have a critical point at $(1, 0)$?