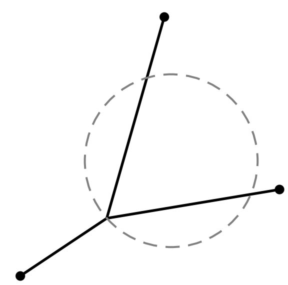
Douglass Houghton Workshop, Section 2, Thu 11/13/25

Worksheet Some Rise By Sin, and Some By Virtue Fall

1. Shortest Network. Last time we used calculus to show that a Λ-shaped network can be improved if the vertex angle is less than 120°.

   (a) Prove that the network to the right is NOT minimizing. You don't need to find the optimal network, just prove that this one can be improved. Hint: consider the portion of the network that is inside the circle.

   (b) What allowed that trick to work? Phrase your answer like this: "Any network which contains ______ can be improved."

   (c) Put it all together, and explain where the soap puts the roundabout.

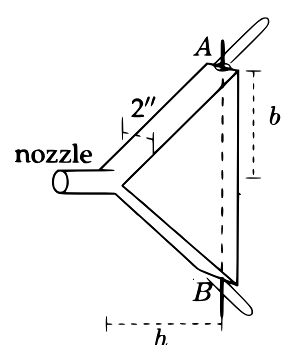
2. (Adapted from a Fall, 2004 Math 115 final) Hao's family spend Thanksgiving eating ramen noodles. The rate at which they eat ramen noodles is given by the function $r(t)$, where t is measured in hours and $r(t)$ is in bowls of noodles/hour. Suppose $t = 0$ corresponds to 10 am, when the binge begins.

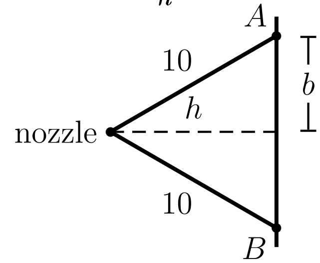
   (a) Write a definite integral that represents the total amount of ramen noodles Hao's family consume between noon and 10 pm.

   (b) If Hao's family's rate of eating ramen noodles is given by $r(t) = 10e^{-t} + 1$, use a left hand sum with three (3) subdivisions to estimate the amount of ramen noodles Hao's family eat in the first four hours of their binge.

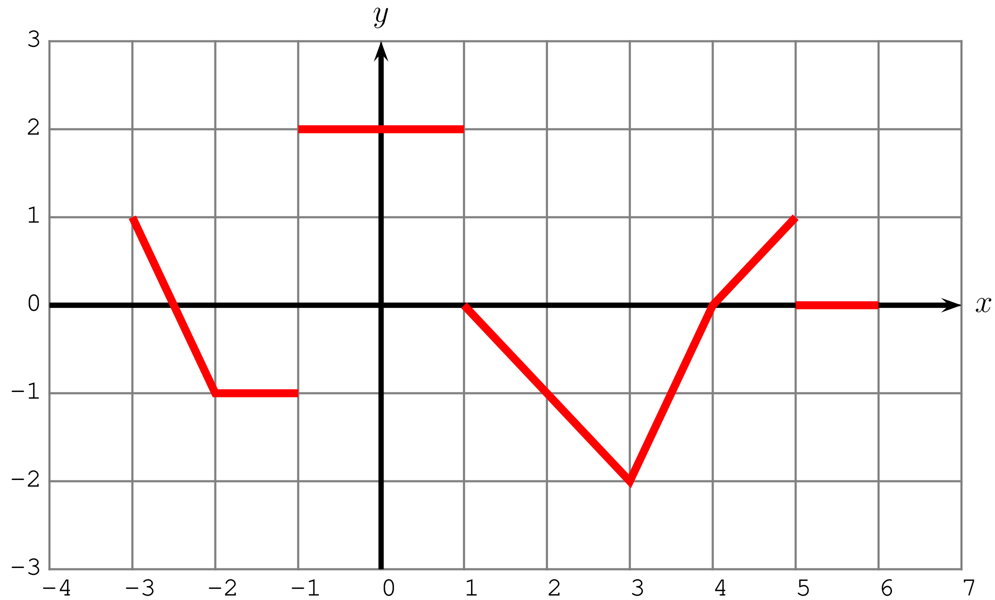
   (c) Should your estimate in part (b) be an underestimate or an overestimate? Explain.

3. (This problem appeared on a Winter, 2008 Math 115 exam) A bellows has a triangluar frame made of three rigid pieces. Two pieces, each 10 inches long, are hinged at the nozzle. They are attached to the third piece at points A and B which can slide, as shown in the diagrams below. (The figures show a 3D sketch of the bellows and a 2D sketch that may be specifically useful to solve the problem.)
   Each piece of the frame is 2 inches wide, so the volume (in cubic inches) of air inside the bellows is equal to the area (in square inches) of the triangluar cross-section above, times 2. Suppose you pump the bellows by moving A downward toward the center at a constant speed of 3 in/sec. (So B moves upwards at the same speed.) What is the rate at which air is being pumped out when A and B are 12 inches apart? (So A is 6 inches from the center of the vertical piece of the frame.)





4. (Fall, 2018) Part of the graph of a piecewise-linear function $r(x)$ is shown below.



The function $h(x)$ is a continuous antiderivative of $r(x)$ with $h(0) = 1$. Sketch the graph of $h(x)$ over the interval $-3 \le x \le 6$. Make sure to pay attention to:

- where h is and is not differentiable.
- where h is increasing/decreasing/constant.
- where h is linear/concave up/concave down.
- the values of $h(-3), h(-2), h(-1), \ldots, h(5), h(6)$.

5. Suppose $\int_4^9 (4f(x) + 7)\,dx = 315$. Find $\int_4^9 f(x)\,dx$.

6. (Winter, 2010) Suppose that the standard price of a round-trip plane ticket from Detroit to Paris, purchased t days after April 30, is $P(t)$ dollars. Assume that P is an invertible function (even though this is not always the case in real life). In the context of this problem, give a practical interpretation for each of the following:

(a) $P'(2) = 55$

(b) $\int_5^{10} P'(t)\,dt$

(c) $P^{-1}(690)$

(d) $\frac{1}{5}\int_5^{10} P(t)\,dt$

7. (Winter, 2012) Consider the family of functions

$$f(x) = ax - e^{bx},$$

where a and b are positive constants.

(a) Any function $f(x)$ in this family has only one critical point. In terms of a and b, what are the x- and y-coordinates of that critical point?

(b) Is the critical point a local maximum or a local minimum? Justify your answer with either the first-derivative test or the second-derivative test.

(c) For which values of a and b will $f(x)$ have a critical point at $(1, 0)$?