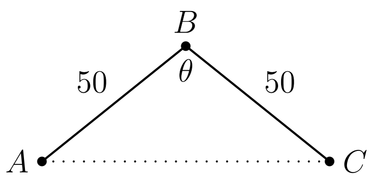
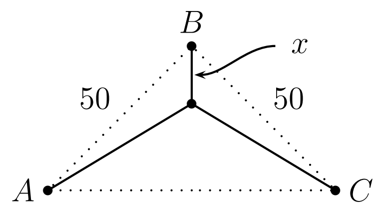
Douglass Houghton Workshop, Section 2, Tue 11/11/25

Worksheet Romeo, Romeo, Wherefore Art Thou Romeo?

1. Shortest Network. So far we've looked at the case where the cities are at the corners of an isosceles triangle like the one below. We know:



- When angle B is 70° or 90°, it is possible to improve upon the Λ-shaped network shown by building a roundabout south of B and connecting it to all three cities.
- However, when B is 150°, the Λ is better than all possible ⅄'s.



(a) Suppose the measure of angle B is θ. Use the law of cosines to write a formula for the length of the ⅄-shaped network to the right, in terms of θ and x.

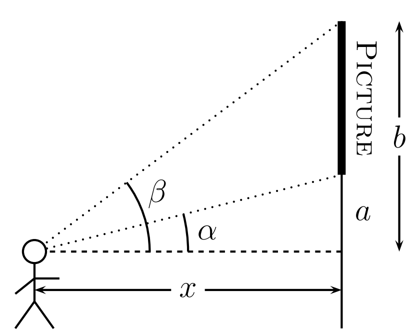
(b) Call that function $L_\theta(x)$. Put your calculator in degrees mode and plot $L_{70}(x)$, $L_{90}(x)$, and $L_{150}(x)$, for x from 0 to 50. Put the graphs on the board.

(c) The shapes seem about the same. What can you see in the graphs that explains the fact that in two cases the Λ can be improved, and in the other it can't? (Remember the Λ is $x = 0$.)

(d) Use calculus to figure out which Λ's can be improved, and which can't. State the result in the form: "Any Λ-shaped network with an angle smaller than ______ can be improved".

(e) This is for those who like to compute and simplify. Show that the function $L_\theta(x)$ defined above is always concave up, by finding and simplifying its second derivative.

2. Annie and Masuma, both accomplished artists, collaborate on a large drawing that is mounted on a wall in a gallery. Its bottom is a feet above eye level, and its top is b feet above eye level. If you stand far away from the wall, you can't see the picture well. But if you stand close to the wall, you can't see well either! So the question is: how far from the wall should you stand in order to have the best view?

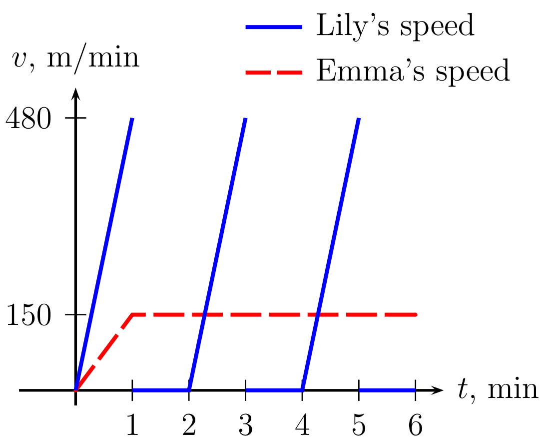


(a) Let α and β be the angles between eye level and the bottom and top of the picture, as shown. x is your distance from the wall. Find α and β in terms of x, a, and b.

(b) $\beta - \alpha$ is the angle that the picture takes up in your field of vision. So find the value of x that maximizes $\beta - \alpha$.

3. (Adapted from a Winter 2009 Math 115 Exam.) Emma and Lily, after much friendly trash talk about who's the fastest runner, decide to have a race. The two employ very different approaches.

- Emma takes the first minute to accelerate to a slow and steady pace which she maintains through the remainder of the race.
- Lily, on the other hand, spends the first minute accelerating to faster and faster speeds until she's exhausted and has to stop and rest for a minute—and then she repeats this process until the race is over. The graph below shows their speeds (in meters per minute), t minutes into the race. (Assume that the pattern shown continues for the duration of the race.)



(a) What is Emma's average speed over the first two minutes of the race? What is Lily's?

(b) Lily immediately gets ahead of Emma at the start of the race. How many minutes into the race does Emma catch up to Lily for the first time?

(c) Draw graphs of Lily's and Emma's positions at time t. Be as precise as possible.

(d) If the race is 720 meters total, who wins? What if it's 721 meters?

4. (This problem appeared on a Winter, 2008 Math 115 exam) A bellows has a triangluar frame made of three rigid pieces. Two pieces, each 10 inches long, are hinged at the nozzle. They are attached to the third piece at points A and B which can slide, as shown in the diagrams below. (The figures show a 3D sketch of the bellows and a 2D sketch that may be specifically useful to solve the problem.)
Each piece of the frame is 2 inches wide, so the volume (in cubic inches) of air inside the bellows is equal to the area (in square inches) of the triangluar cross-section above, times 2. Suppose you pump the bellows by moving A downward toward the center at a constant speed of 3 in/sec. (So B moves upwards at the same speed.) What is the rate at which air is being pumped out when A and B are 12 inches apart? (So A is 6 inches from the center of the vertical piece of the frame.)