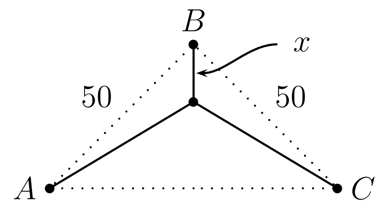
Douglass Houghton Workshop, Section 2, Thu 11/06/25

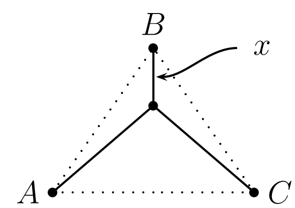
Worksheet Quoth the Raven, "Nevermore"

1. We've been working on the problem of finding the shortest road network between three cities in the plane.

   In the case we considered, the three cities were at the corners of a 45°-45°-90° triangle with legs 50 miles long. The simplest idea is to just build roads along the legs; that makes a network of length 100. But by constructing a ⅄-shaped network like the one at the right, we found

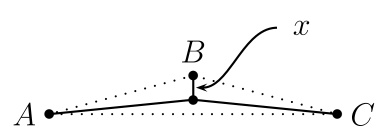
   

   - The length of the network is $x + 2\sqrt{2500 - 100x\cos(45) + x^2}$.
   - We can improve from the simple 2-road solution ($x = 0$, length $= 100$) by increasing x. For instance, when $x = 10$, the network has a length of about 97.

   (a) Consider the case where the triangle is still isosceles and the legs still have length 50, but the angle at B is 70°. Write a formula for the length of the network.

   (b) Can you find a value of x which beats the 2-road solution ($x = 0$, length $= 100$)?

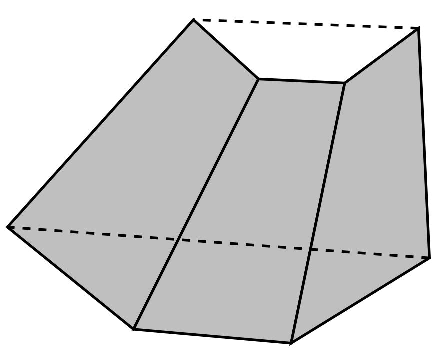
   

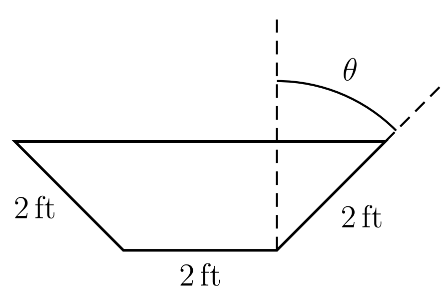
   (c) Now suppose the vertex angle is very obtuse—say 150°. Find a formula for the length of the network.

   (d) Can you beat the 2-road solution in this case?

   (e) Suppose the vertex angle is θ. Write a formula for the length of the network.

2. A trough, as shown below, is to be made with a base that is 2 feet wide and 10 feet long. The sides of the trough are also 2 feet wide by 10 feet long, and are to be placed so they make an angle θ with the vertical.

   (a) What is the area, in terms of θ, of a cross section of the trough perpendicular to its long side? What is the volume of the trough?

   (b) What angle θ will give the trough the largest volume, and what is that volume? [Hint: you can always replace $\cos^2(\theta)$ with $1 - \sin^2(\theta)$.]

3. November is National Novel Writing Month, and many people around the country attempt to complete a first draft of a novel in the course of the month. One of them is Mark's friend Chris. At the end of every day November she uploaded her manuscript to a website (`nanowrimo.org`), which counts how many words she has written. Here are her counts, rounded to the nearest hundred:

Nov.	Count	Nov.	Count	Nov.	Count	Nov.	Count	Nov.	Count
1	1700	5	8300	9	9400	13	14800	17	23300
2	3600	6	8300	10	9400	14	17000	18	24700
3	5800	7	8700	11	11800	15	20100	19	26600
4	7500	8	8700	12	13800	16	21300	20	26700

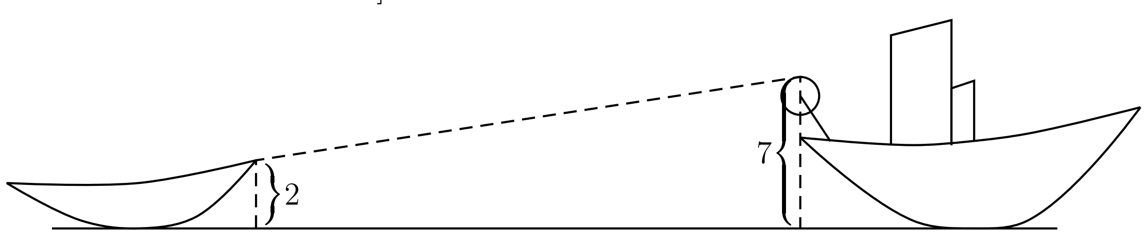
(a) Let x be the time in days since the start of November, and let $W(x)$ be the total number of words Chris has written at time x. Assume that each day Chris writes at a steady rate, from midnight to midnight. (But different rates for different days.) Draw a graph of $W(x)$ for the second week (x from 7 to 14).

(b) Let $w(x)$ be the derivative of $W(x)$. Draw a graph of $w(x)$ for x from 7 to 14.

(c) Now consider the function $F(t)$, which is the area between the line $x = 7$, the line $x = t$, the x-axis, and the graph of $w(x)$. Make a table of values showing $F(7), F(8), \ldots, F(14)$. What do you notice? Explain this result.

4. (Adapted from a Fall, 2006 Math 115 Final Exam.) Morgan is surfing off Maui when she is swept away from the beach by a rogue current. Fortunately a tugboat is passing by, and Morgan hails it for a tow back to shore.

The tugboat captain throws Morgan a line, and she attaches it to her board 2 meters above the water line. The other end of the cable is attached to a wheel of radius 0.5 meters sitting on the back of the tugboat. The top of the wheel is 7 meters above the water, and turns at a constant rate of 1 revolution per second. [See the figure below—not drawn to scale.]



(a) At what rate is the length of the cable between the board and the tugboat changing?

(b) How fast is Morgan's board being pulled forward when it is 10 meters away from the tugboat?