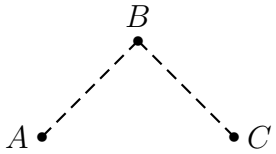
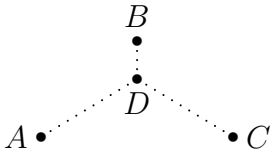
Douglass Houghton Workshop, Section 2, Tue 11/04/25

Worksheet Past is Prologue

1. The three cities in the pictures below are at the corners of a 45°-45°-90° triangle whose legs are 50 miles long. The three mayors, working together, would like to build roads between them in such a way that there is a way to get from any one city to any other city.





(Say, A is Ann Arbor, B is Flint, and C is Port Huron.) The first, simple proposal (on the left) is to build a road from A to B and another from B to C. That would certainly work. But roads are expensive, and one of the mayors (who, luckily, studied calculus) proposes building roads from A and C to a point D just south of B, then building a road north from there to B.

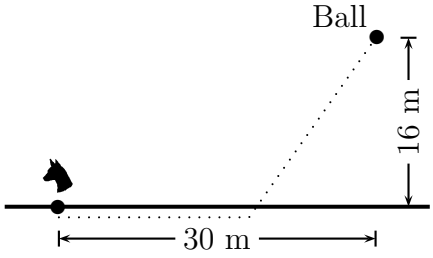
   (a) Let x be the length of the north-south road in the second proposal. What does it mean if $x = 0$?

   (b) Calculate the total length of the new network in terms of x. Hint: "Law of cosines".

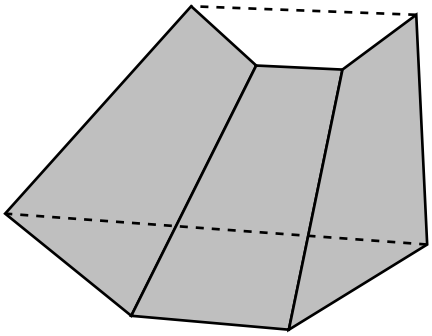
   (c) Can you find a value of x which will produce a shorter network than the simple proposal?

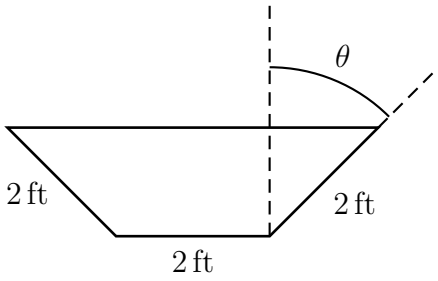
2. As we know, Juliana has 5 dogs: Louise, Aspen, Bentley, Diamond, and Ellie. Bentley likes to fetch, and one day he and Juliana are walking along the Grand River in Lansing. Juliana throws a ball 30 meters down the beach and 16 meters out into the water.

Bentley, being practical, wants to get to the ball as quickly as possible. The thing is that he can run faster than he can swim; his running speed on the beach is 9 meters per second, and he can swim 3 meters per second. How should Bentley (who has an intuitive notion of calculus) get to the ball?



3. A trough, as shown below, is to be made with a base that is 2 feet wide and 10 feet long. The sides of the trough are also 2 feet wide by 10 feet long, and are to be placed so they make an angle θ with the vertical.





(a) What is the area, in terms of θ, of a cross section of the trough perpendicular to its long side? What is the volume of the trough?

(b) What angle θ will give the trough the largest volume, and what is that volume? [Hint: you can always replace $\cos^2(\theta)$ with $1 - \sin^2(\theta)$.]

4. November is National Novel Writing Month, and many people around the country attempt to complete a first draft of a novel in the course of the month. One of them is Mark's friend Chris. At the end of every day November she uploaded her manuscript to a website (`nanowrimo.org`), which counts how many words she has written. Here are her counts, rounded to the nearest hundred:

Nov.	Count	Nov.	Count	Nov.	Count	Nov.	Count	Nov.	Count
1	1700	5	8300	9	9400	13	14800	17	23300
2	3600	6	8300	10	9400	14	17000	18	24700
3	5800	7	8700	11	11800	15	20100	19	26600
4	7500	8	8700	12	13800	16	21300	20	26700

(a) Let x be the time in days since the start of November, and let $W(x)$ be the total number of words Chris has written at time x. Assume that each day Chris writes at a steady rate, from midnight to midnight. (But different rates for different days.) Draw a graph of $W(x)$ for the second week (x from 7 to 14).

(b) Let $w(x)$ be the derivative of $W(x)$. Draw a graph of $w(x)$ for x from 7 to 14.

(c) Now consider the function $F(t)$, which is the area between the line $x = 7$, the line $x = t$, the x-axis, and the graph of $w(x)$. Make a table of values showing $F(7), F(8), \ldots, F(14)$. What do you notice? Explain this result.