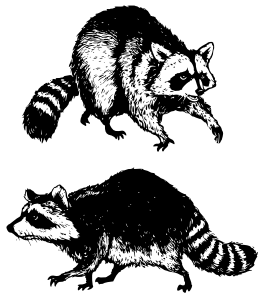
**Douglass Houghton Workshop, Section 2, Thu 10/23/25**

# Worksheet Now is the Winter of our Discontent



1. Annie is studying a population of raccoons near Memphis. Suppose that the population changes according to the rule:

$$P(n+1) = 1.5P(n) - 200$$

where $P(0)$ is the population in 2025, $P(1)$ is the population 1 year later, etc. ($P$ is measured in raccoons.)

   (a) Make up a (short) story about raccoons that explains the formula above.

   (b) Suppose $P = 320$ in 2025. What will happen in the long run?

   (c) Suppose instead that $P = 440$ in 2025. Now what happens?

   (d) A population is in **equilibrium** if it stays the same from year to year. Is there an equilibrium number for this population?

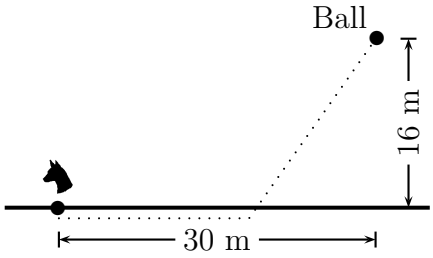
   (e) Explain these results pictorially by drawing the graphs of $y = x$ and $y = 1.5x - 200$. Start at $(320, 320)$, go down to the other graph, and then over to $y = x$. That's the new population. Repeat. Then start at 440.

2. Repeat the last problem, but for the rule

$$P(n+1) = .75P(n) + 125.$$

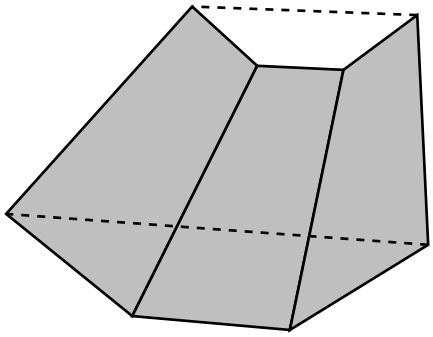
3. As we know, Juliana has 5 dogs, Juliana has 5 dogs: Louise, Aspen, Bentley, Diamond, and Ellie. Bentley likes to fetch, and one day he and Juliana are walking along the Grand River in Lansing.. Juliana throws a ball 30 meters down the beach and 16 meters out into the water.

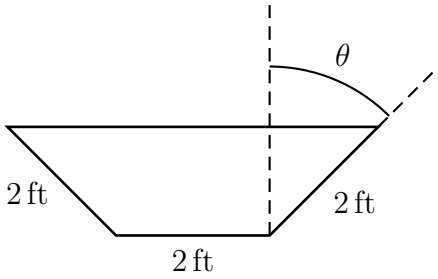
   Bentley, being practical, wants to get to the ball as quickly as possible. The thing is that he can run faster than he can swim; his running speed on the beach is 9 meters per second, and he can swim 3 meters per second. How should Bentley (who has an intuitive notion of calculus) get to the ball?




4. Let $f^{(n)}(x)$ denote the $n$th derivative of $f$. If $f(x) = e^{-2x}$, find $f^{(531)}(x)$. Is $f^{(531)}(x)$ increasing or decreasing? Concave up or concave down? Try graphing $f^{(531)}$ without your calculator, then check with the calculator.

5. A trough, as shown below, is to be made with a base that is 2 feet wide and 10 feet long. The sides of the trough are also 2 feet wide by 10 feet long, and are to be placed so they make an angle $\theta$ with the vertical.






   (a) What is the area, in terms of $\theta$, of a cross section of the trough perpendicular to its long side? What is the volume of the trough?

   (b) What angle $\theta$ will give the trough the largest volume, and what is that volume? [Hint: you can always replace $\cos^2(\theta)$ with $1 - \sin^2(\theta)$.]

6. (This problem appeared on a Winter 2007 Math 115 exam) Suppose $f$ and $g$ are differentiable functions with values given by the table below.

   (a) If $h(x) = f(x)g(x)$, find $h'(3)$.

   (b) If $j(x) = \dfrac{(g(x))^3}{f(x)}$, find $j'(1)$.

   (c) If $d(x) = x \ln\left(e^{f(x)}\right)$, find $d'(3)$.

   (d) If $t(x) = \cos(g(x))$, find $t'(1)$.

| $x$ | $f(x)$ | $g(x)$ | $f'(x)$ | $g'(x)$ |
|---|---|---|---|---|
| 1 | 2 | 9 | −3 | 7 |
| 3 | 4 | 11 | 15 | −19 |