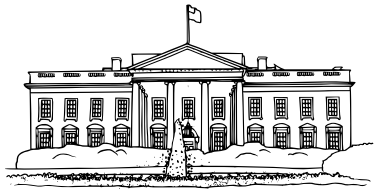
Douglass Houghton Workshop, Section 2, Tue 09/30/25

Worksheet Koala

1. We still have this $1/z$ scale model of the White House, which we plan on blowing up. We want to decide what speed to run the film at, so that when we slow it down to 24 frames per second, we get a realistic explosion.



   (a) Last time we showed that an object will fall $16t^2$ feet in t seconds. So how long does it take for an object to fall off the real white house, which is H feet tall? How many frames will that be, if we film it at 24 frames per second and show it at the same speed?

   (b) How long does it take an object to fall off the top of the model?

   (c) How many frames per second should you film to get the right number of frames to make it look like the model is full-sized?

2. Consider a mirror in the shape of the graph of $y = \pm\sqrt{4x}$.

   (a) Draw the mirror (make it big). What shape is it?

   (b) Draw a light ray travelling leftward along the line $y = -b$, where b is some positive number (making $-b$ negative). At what point P does the ray hit the mirror?

   (c) Find, in terms of b, the slope of the tangent to the mirror at P.

   (d) The *normal* to a curve at a point is the line through that point which is perpendicular to the tangent line. Find the slope of the normal to the mirror at P, and draw both the normal and tangent lines on your graph.

   (e) Suppose a line makes an angle θ with the positive x-axis. What is the slope of the line?

   (f) Let θ be the angle the normal to the mirror at P makes with the light ray $y = -b$. Can you write θ in terms of b? Hint: Use (2d) and (2e).

   To be continued...

3. Suppose $h(x)$ is a continuous function defined for all real numbers x. The derivative and second derivative of $h(x)$ are given by

$$h'(x) = \frac{2x}{3(x^2-1)^{2/3}} \qquad \text{and} \qquad h''(x) = -\frac{2(x^2+3)}{9(x^2-1)^{5/3}}.$$

   (a) Find the all critical points and local extrema of $h(x)$. Use calculus to classify the critical points and justify your answers, and be sure to show enough evidence to demonstrate that you have found all local extrema.

   (b) Find all inflection points of $h(x)$, and justify that you've found them all.

4. Yusuf drives on the New York Thruway for A Model UN Conference. He enters at Angola, where a camera takes a picture of the car's license plate. The picture is processed and the licensce plate stored in a database, along with the time the picture was taken. Later Yusuf exits at Albany, where another picture is taken. A week afterward, he receives a bill in the mail along with a speeding citation, stating that he was going exactly 75 mph at some point on his trip. How does the Mean Value Theorem allow the authorities to be sure that happened?

5. (An old team homework problem.) Let $f(x) = x^2 - 2x + 13$ and $g(x) = -x^2 - 2x - 5$.

   (a) Draw $y = f(x)$ and $y = g(x)$ on the same set of axes. How many lines are tangent to both graphs?

   (b) Find the equations of those lines.

6. Molecules absorb far-infrared radiation because its excites their rotation. The absorption coefficient a of a given liquid varies with the frequency ω of the radiation according to

$$a(\omega) = \frac{10}{\omega^2 - 2c\omega + 125}$$

   where c is some constant $(0 \leq c \leq 11)$.

   (a) For what value of the frequency ω is the absorption a maximum?

   (b) Graph $a(\omega)$ for $c = 11$. How would you describe the shape of this graph?

   [Note: with appropriate parameters this function describes the shapes of the lines in many kinds of spectroscopy].

7. (This problem appeared on a Winter, 2004 Math 115 exam. Really!) While exploring an exotic spring break location, you discover a colony of geese who lay golden eggs. You bring 20 geese back with you. Suppose each goose can lay 294 golden eggs per year. You decide maybe 20 geese isn't enough, so you consider getting some more of these magical creatures. However, for each extra goose you bring home there are less resources for all the geese. Therefore, for each new goose the amount of eggs produced will decrease by 7 eggs per goose per year. How many more geese should you bring back if you want to maximize the number of golden eggs per year laid?