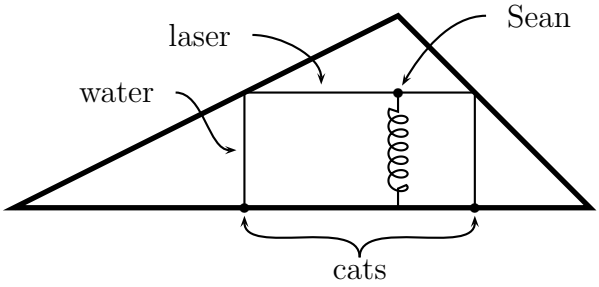
Douglass Houghton Workshop, Section 2, Thu 09/18/25

Worksheet Harbinger of Things to Come

1. Explain how to use two rulers to add numbers.

2. Explain how a slide rule is able to multiply two numbers. This picture may be helpful:

3. Sean is being raised on a giant spring-loaded platform right under the peak of a triangular room. Laser beams are being emitted from his head, parallel to the ground, until they hit the walls. Where they hit the walls, drops of water fall down, then land in the mouths of two cats. As Sean goes up, the cats follow the drops toward the base of the platform.



   (a) Let $h(t)$ be Sean's height at time t, and let $w(t)$ be the distance between the two cats. Are they continuous functions? Is $h(t) - w(t)$ a continuous function?

   (b) When t is close to 0 (so Sean's head has just come through the floor), what can you say about $h(t) - w(t)$?

   (c) Later on, when Sean is near the end of his journey and about to hit the top, what can you say about $h(t) - w(t)$?

   (d) Use the Intermediate Value Theorem to show that at some time the distance between the cats is the same as Sean's height off the floor.

4. Why is it necessary to define the derivative in terms of a limit? Draw a picture that describes how the derivative is the limit of the slopes of some lines.

5. Prove that it's possible to make a fair five-sided die. Rules:

   (a) All sides must be flat,

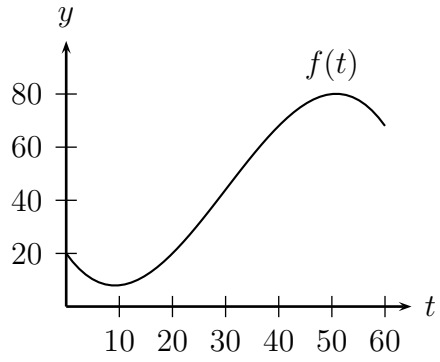
   (b) It must be equally likely to land on all sides, and

   (c) No handles (ala a dreidel).

6. The *power rule for derivatives* says that if $f(x) = x^n$, then $f'(x) = nx^{n-1}$. Use the definition of the derivative to prove it for the case where n is a positive integer. Hint: Pascal's triangle.

7. (Fall, 2012) Your pet bird is flying in a straight path toward you and away from you for a minute. After t seconds, she is $f(t)$ feet away from you, where

$$f(t) = \frac{-t(t-20)(t-70)}{500} + 20, \qquad 0 \le t \le 60.$$



(a) Without doing any calculations, determine which is greater: the average velocity of the bird over the entire minute, or her instantaneous velocity after 30 seconds. Explain, referring to the graph.

(b) Calculate the exact value of the average velocity of the bird over the entire minute.

(c) Write an explicit expression for the velocity of the bird at time t using the limit definition of velocity. Final answers containing the letter f will receive no credit. Do not evaluate your expression.

(d) After a minute, you scare the bird, and she flies away at 9 feet/sec. Write a formula for a continuous function $f(t)$ describing the distance between you and the bird for $0 \le t \le 180$.

8. (Winter, 2010) Suppose $W(h)$ is an invertible function which tells us how many gallons of water an h-foot tall oak tree uses on a hot summer day.

(a) Give practical interpretations of $W(50)$ and $W^{-1}(40)$.

(b) Suppose that an average oak tree is A feet tall and uses G gallons of water on a hot summer day. Answer in terms of W, A, and G:

i. A farmer has 25 oak trees, and each one is 10 feet taller than an average oak tree. How much water will they use on a hot summer day?

ii. The farmer also has some oak trees which use 5 fewer gallons of water on a hot summer day than an average oak tree does. How tall are his trees?

9. (This problem appeared on a Winter, 2014 Math 115 exam) Find all vertical and horizontal asymptotes of the graph of

$$g(x) = \frac{k(x-a)(x-b)}{(x-a)(x-c)^2}$$

where a, b, c, and k are constants with $a < b < c < k \neq 0$.