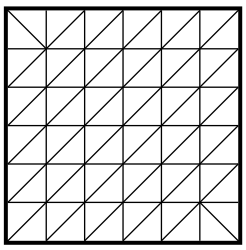
Douglass Houghton Workshop, Section 2, Tue 09/16/25

Worksheet Go Forth and Multiply

1. Cake! We had a breakthrough last time. We found that for 6 people we could cut the cake up into 72 little triangles, and give each person 4 outside triangles and 8 inside triangles.



   (a) Draw the picture above on the board. Then decide which pieces to give to person number 1, and write "1" in each. It would be great if they were all connected.

   (b) Do the same for persons 2–6. Can you simplify so that we have to make as few cuts as possible?

   (c) Generalize to any number of people (especially 11).

2. Explain how to use two rulers to add numbers.

3. Explain how a slide rule is able to multiply two numbers. This picture may be helpful:

4. Why is it necessary to define the derivative in terms of a limit? Draw a picture that describes how the derivative is the limit of the slopes of some lines.

5. The *power rule for derivatives* says that if $f(x) = x^n$, then $f'(x) = nx^{n-1}$. Use the definition of the derivative to prove it for the case where n is a positive integer. Hint: Pascal's triangle.

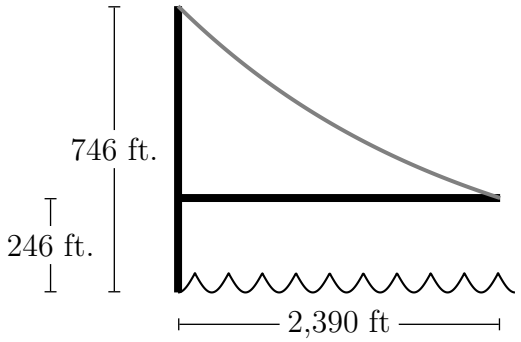
6. Prove that it's possible to make a fair five-sided die. Rules:

   (a) All sides must be flat,

   (b) It must be equally likely to land on all sides, and

   (c) No handles (ala a dreidel).

7. (This question appeared on a Fall, 2008 Math 115 exam.) San Francisco's famous Golden Gate bridge has two towers which stand 746 ft. above the water, while the bridge itself is only 246 ft. above the water. The last leg of the bridge, which connects to Marin County, is 2,390 ft. long. The suspension cables connecting the top of the tower to the mainland can be modeled by an exponential function. Let $H(x)$ be the function describing the height above the water of the suspension cable as a function of x, the horizontal distance from the tower.



(a) Find a formula for $H(x)$.

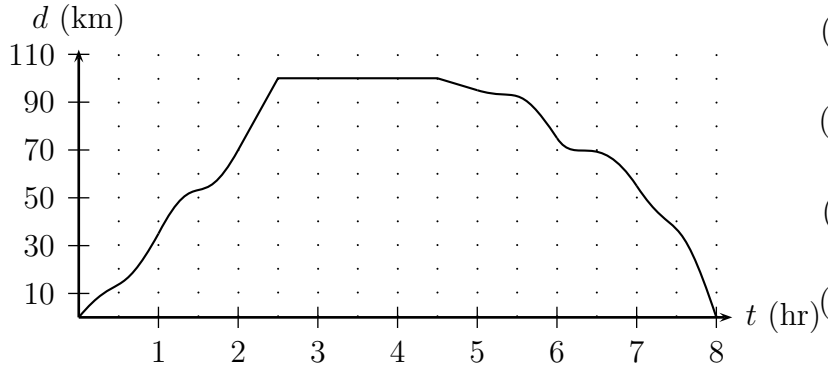
(b) The engineers determined that some repairs are necessary to the suspension cables. They climb up the tower to 400 ft above the bridge, and they need to lay a horizontal walking board between the tower and the suspension cable. How long does the walking board need to be to reach the cable?

8. (This problem appeared on a Winter, 2014 Math 115 Exam.) The air in a factory is being filtered so that the quantity of a pollutant, P (in mg/liter), is decreasing exponentially. Suppose t is the time in hours since the factory began filtering the air. Also assume 20% of the pollutant is removed in the first five hours.

(a) What percentage of the pollutant is left after 10 hours?

(b) How long is it before the pollution is reduced by 50%?

9. (This problem appeared on a Winter, 2014 Math 115 Exam) A ship's captain is making a round trip voyage between two ports. The ship sets sail from Port Jackson at noon, arrives at Port Kembla some time later, waits there for a while, and then returns to Port Jackson. Let $s(t)$ be the ship's distance, in kilometers, from its starting point of Port Jackson, t hours after noon. A graph of $d = s(t)$ is shown below.



(a) How far is Port Kembla from Port Jackson?

(b) How long does the ship wait in Port Kembla?

(c) What is the ship's average speed during the return trip?

(d) Estimate the ship's instantaneous velocity at 1pm.

(e) Sometime after 5pm, there is a time when the ship's instantaneous velocity is 0 km/hr. When does this occur?