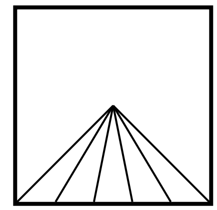
Douglass Houghton Workshop, Section 2, Thu 09/11/25

Worksheet Fluffernutter

1. Cake! We had many successful ideas last time. One involved cutting a portion of the cake into triangles like the picture shown here. How do you cut the triangles so that the pieces have the same amount of cake and the same amount of frosting? Use that to find a solution for 12, 16, 20 people. Now find a solution for any multiple of 4. Now make that into a soultion for any number of people.



2. We're still sitting in this cabin, trying to figure out the temperature in Fahrenheit. We have a Celsius thermometer, $33\frac{1}{3}$ RPM record player, a roll of duct tape, a Beatles album from 1967, a stopwatch, and some very cold feet. We know that if c is the temperature we read on the Celsius thermometer, then $f = \frac{9}{5}c + 32$ is the temperature in Fahrenheit. We need to convert from Celsius to Fahrenheit without multiplying or dividing.

3. Bankers and financial advisors use what they call the **Rule of 70**. It says:

   > If you invest money at annual interest rate r percent, it will take about $70/r$ years for your money to double.

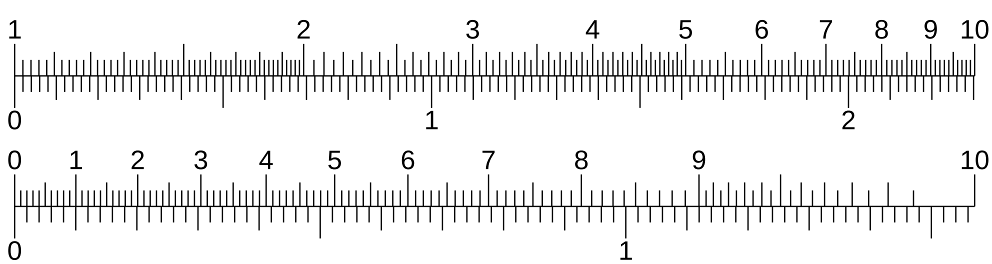
   (So, for instance, \$500 invested at 5% interest will be worth \$1000 in about about 14 years, because $14 = 70/5$.)

   (a) Explain why the Rule of 70 works, and what assumptions you need to make it work. Hint: recall what we learned from Michael Phelps's towel:

   $$\lim_{n\to\infty}\left(1+\frac{T}{n}\right)^n = e^T.$$

   (b) Devise a similar rule for the time it takes your money to triple at r% interest.

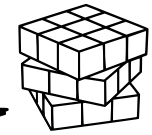
4. What's the deal with these pictures? What are they good for?



5. We've all seen 6-sided dice, and we presume they are "fair", in the sense that all 6 sides are equally likely to land on the bottom. Can you construct a fair 4-sided die? How about an 8-sided die? What other sizes are possible?

6. Sean has noticed that his tastes changed over the last year. A year ago he spent about 15 hours a week doing gymnastics, and 10 hours solving the Rubik's Cube. Gradually school took over his life, and though there have been some ups and downs in his schedule, the general trend is that he's spent less time per week on both. Now, 52 weeks later, he spends only 3 hours a week doing gymnastics and 5 hours a week solving the Rubik's Cube.

   Let $G(t)$ be the number of hours Sean spent doing gymnastics in week t, and let $R(t)$ be the number of hours he spent solving the Rubik's Cube. Assume $G(t)$ and $R(t)$ are continuous functions of time.



   (a) What does it mean for a function to be continuous?

   (b) Are $G(t) + R(t)$, $G(t) - R(t)$, and $G(t)R(t)$ continuous?

   (c) Use one of the functions in part (a) together with the Intermediate Value Theorem to show that at some time in the last year, Sean was spending the same amount of time doing gymnastics and solving the Rubik's Cube.

7. (This problem is adapted from a Fall, 2015 Math 115 Exam) Katie is jumping rope while Tony runs a stopwatch. There is a piece of tape around the middle of the rope. When the rope is at its lowest, the piece of tape is 2 inches above the ground, and when the rope is at its highest, the piece of tape is 68 inches above the ground. The rope makes two complete revolutions every second. When Tony starts her stopwatch, the piece of tape is halfway between its highest and lowest points and moving downward. The height H (in inches above the ground) of the piece of tape can be modeled by a sinusoidal function $K(t)$, where t is the number of seconds displayed on Tony's stopwatch.

   (a) Sketch a well-labeled graph of two periods of $K(t)$ beginning at $t = 0$.

   (b) Find a formula for $K(t)$.

   (c) Now Tony takes a turn at jumping. Katie resets the stopwatch and starts it over again. Let $T(w)$ be the height (in inches above the ground) of the piece of tape when Katie's stopwatch says w seconds. A formula for $T(w)$ is $T(w) = 41 + 38\cos(2\pi w)$. Katie is 60 inches tall. For how long (in seconds) during each revolution of the rope is the piece of tape higher than the top of Katie's head? (Assume Katie is standing straight while watching the stopwatch.)

8. (This problem appeared on a Winter, 2013 Math 115 Exam.) The air in a factory is being filtered so that the quantity of a pollutant, P (in mg/liter), is decreasing exponentially. Suppose t is the time in hours since the factory began filtering the air. Also assume 20% of the pollutant is removed in the first five hours.

   (a) What percentage of the pollutant is left after 10 hours?

   (b) How long is it before the pollution is reduced by 50%?