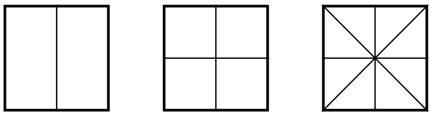
Douglass Houghton Workshop, Section 2, Tue 09/09/25

Worksheet Elephant

1. Suppose you bake a square cake, 10 inches on a side and 2 inches high. You frost it on the top and all four sides (but not the bottom). We want to split the cake among n people, and we want everyone to get equal shares of cake and frosting. So far we've figured out how to do it for $n = 2$, $n = 4$, and $n = 8$:

   We had a number of other ideas too. What other numbers of people can you accommodate? Explain exactly how to cut the cake and why it is fair.

2. We're still sitting in this cabin, trying to figure out the temperature in Fahrenheit. We have a Celsius thermometer, $33\frac{1}{3}$ RPM record player, a roll of duct tape, a Beatles album from 1967, a stopwatch, and some very cold feet. We know that if c is the temperature we read on the Celsius thermometer, then $f = \frac{9}{5}c + 32$ is the temperature in Fahrenheit. We need to convert from Celsius to Fahrenheit without multiplying or dividing.

3. *Michael Phelps: The Sequel* Michael Phelps took all the money (let's say it's 2 million dollars) he got for endorsing Speedo, Visa, Subway, Frosted Flakes, and Head & Shoulders shampoo, and put it into a bank. The bank has several accounts available. For each, write an expression for how much Michael will have t years from now.

   (a) 6% interest, compounded annually.

   (b) 5% interest, compounded monthly.

   (c) 4% interest, compounded daily.

   (d) interest rate r, compounded n times per year.

   The bank also has something called "continuously compounded interest", which means that the number of compoundings per year is really really large. Write a limit expression for the amount of money he'll have if he gets interest rate r, compounded continuously.

4. Bankers and financial advisors use what they call the **Rule of 70**. It says:

   > If you invest money at annual interest rate r percent, it will take about $70/r$ years for your money to double.

   (So, for instance, \$500 invested at 5% interest will be worth \$1000 in about about 14 years, because $14 = 70/5$.)

   (a) Explain why the Rule of 70 works, and what assumptions you need to make it work. Hint: recall what we learned from Michael Phelps's towel:

   $$\lim_{n\to\infty}\left(1+\frac{T}{n}\right)^n = e^T.$$

   (b) Devise a similar rule for the time it takes your money to triple at r% interest.

5. We've all seen 6-sided dice, and we presume they are "fair", in the sense that all 6 sides are equally likely to land on the bottom. Can you construct a fair 4-sided die? How about an 8-sided die? What other sizes are possible?

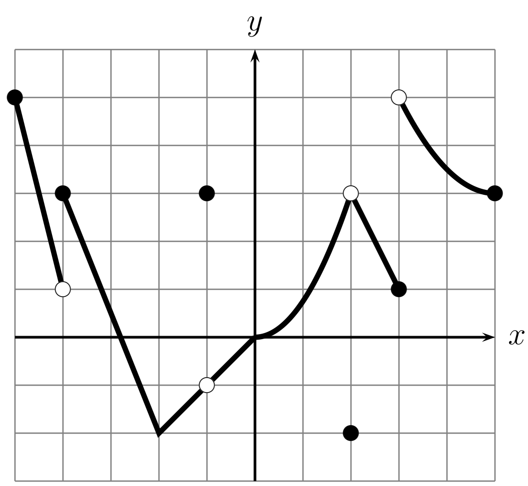
6. (This problem is adapted from a Fall, 2015 Math 115 Exam) Katie is jumping rope while Tony runs a stopwatch. There is a piece of tape around the middle of the rope. When the rope is at its lowest, the piece of tape is 2 inches above the ground, and when the rope is at its highest, the piece of tape is 68 inches above the ground. The rope makes two complete revolutions every second. When Tony starts her stopwatch, the piece of tape is halfway between its highest and lowest points and moving downward. The height H (in inches above the ground) of the piece of tape can be modeled by a sinusoidal function $K(t)$, where t is the number of seconds displayed on Tony's stopwatch.

   (a) Sketch a well-labeled graph of two periods of $K(t)$ beginning at $t = 0$.

   (b) Find a formula for $K(t)$.

   (c) Now Tony takes a turn at jumping. Katie resets the stopwatch and starts it over again. Let $T(w)$ be the height (in inches above the ground) of the piece of tape when Katie's stopwatch says w seconds. A formula for $T(w)$ is $T(w) = 41 + 38\cos(2\pi w)$. Katie is 60 inches tall. For how long (in seconds) during each revolution of the rope is the piece of tape higher than the top of Katie's head? (Assume Katie is standing straight while watching the stopwatch.)

7. (From a Fall, 2017 Math 115 Exam.) The graph of $y = Q(x)$ is shown. The gridlines are one unit apart.



   (a) On which of the following intervals is $Q(x)$ invertible?

   $[-4, -1]$ $[-2, 3]$ $[2, 5]$ $[-2, 2]$

   (b) Find the following limits. Write "NI" if there you don't have enough information and "DNE" if the limit doesn't exist.

   i. $\lim_{x \to -1} Q(x)$ ii. $\lim_{w \to 2} Q(Q(w))$

   iii. $\lim_{h \to 0} \dfrac{Q(-3+h) - Q(-3)}{h}$ iv. $\lim_{x \to \infty} Q\left(\dfrac{1}{x} + 3\right)$ v. $\lim_{x \to \frac{1}{3}} xQ(3x - 5)$

   (c) For which values of $-5 < x < 5$ is the function $Q(x)$ <u>not</u> continuous?

   (d) For which values of $-5 < p < 5$ is $\lim_{x \to p^-} Q(x) \neq Q(p)$?