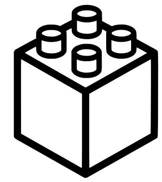
Douglass Houghton Workshop, Section 2, Tue 09/02/25

Worksheet Chocolate Frosting

1. As we know, Abdarrahman built a Lego Technic Lamborghini. Since then he has acquired many more lego sets. He has lego sets of the Millenium Falcon, a Tyrranosaurus Rex, the Titanic, and many more. Currently he has 40 lego sets. He'd really rather leave lego behind, but friends and family keep giving him new lego sets every year.



   Write formulas for the number of lego sets Abdarrahman will have t years from now, under the following conditions:

   (a) Abdarrahman receives 5 new lego sets every year.

   (b) In year t Abdarrahman receives one new lego set for each two lego sets he had in year $t-1$.

   (c) Abdarrahman receives 1 lego set next year, 2 the year after that, 3 the year after that, etc.

2. Last time we found formulas for Michael Phelps' dampness after regular towelling and split towelling. Assuming Michael's body is $1\,\text{m}^2$, the towel is $T\,\text{m}^2$, and he starts with 1 liter of water on him, we have

$$\text{wetness after regular toweling} = \frac{1}{1+T}$$
$$\text{wetness after “split” toweling} = \frac{1}{(1+T/2)^2}.$$

   Let's see just how much this "splitting" idea will buy us.

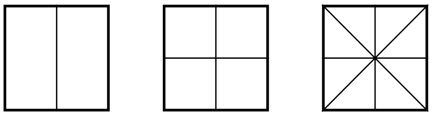
   (a) Suppose Michael splits his towel into 3 parts, and uses all three. How wet will he be? How about if he splits it into n parts?

   (b) Use calculators to fill in the table below with 4-decimal place numbers.

T	$n=1$	$n=10$	$n=100$	$n=1000$	$n=10000$
$1\,\text{m}^2$					
$2\,\text{m}^2$					
$4\,\text{m}^2$					
$\frac{1}{2}\,\text{m}^2$					

   (c) Consider the $1\,\text{m}^2$ towel. How would you describe the effect of dividing it into more and more pieces? For instance, does dividing more always make Michael dryer? Might it make him wetter? Can he get as dry as he might possibly want with that one towel? Does it even matter how big his towel is?

3. Suppose you bake a square cake, 10 inches on a side and 2 inches high. You frost it on the top and all four sides (but not the bottom). We want to split the cake among n people, and we want everyone to get equal shares of cake and frosting. Last time we figured out how to do it for $n = 2$, $n = 4$, and $n = 8$:

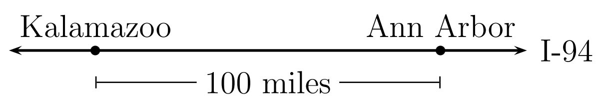


We had a number of other ideas too. What other numbers of people can you accommodate? Explain exactly how to cut the cake and why it is fair.

4. We're still sitting in this cabin, trying to figure out the temperature in Fahrenheit. We have a Celsius thermometer, $33\frac{1}{3}$ RPM record player, a roll of duct tape, a Beatles album from 1967, a stopwatch, and some very cold feet. We know that if c is the temperature we read on the Celsius thermometer, then $f = \frac{9}{5}c + 32$ is the temperature in Fahrenheit. We need to convert from Celsius to Fahrenheit without multiplying or dividing.

5. Kalamazoo is 100 miles west of Ann Arbor along Route 94.

Let $T(x)$ be the temperature in Fahrenheit at a point x miles west of Ann Arbor.



   (a) Define a function A in terms of T so that $A(m)$ is the temperature in Fahrenheit at a point m miles east of Kalamazoo.

   (b) Define a function B in terms of T so that $B(k)$ is the temperature in Fahrenheit at a point k **kilometers** east of Kalamazoo. (1 mile = 1.6 kilometers.)

   (c) Define a function C in terms of T so that $C(k)$ is the temperature in **Celcius** at a point k kilometers east of Kalamazoo.

6. (This problem appeared on a Fall, 2012 Math 115 exam) Suppose p represents the price of a reuben sandwich at a certain restaurant on State St. $R(p)$ represents the number of reubens the restaurant will sell in a day if they charge $\$p$ per reuben.

   (a) What does $R(5.5)$ represent in the context of this situation?

   (b) Assuming R is invertible, what does $R^{-1}(305)$ represent?

   (c) The owner of the restaurant also has a Church St. location. It doesn't get quite as much business, and the owner finds that the State St. store sells 35% more reubens than the Church St. store sells at the same price. Let $C(p)$ be the number of reubens the Church St location sells in a day at a price of $\$p$ each. Write a formula for $C(p)$ in terms of $R(p)$.

   (d) The owner starts doing research on reuben sales at the State Street location; he wants to know how the number of reubens sold is related to price. He finds that every time he raises the price by \$1 per reuben, the number sold in a day decreases by 20%. Let the constant B represent the number of reubens sold in a day at the State Street store if the price of reubens is \$5 each. Write a formula for $R(p)$ involving the constant B. Assume the domain of R is $1 \leq p \leq 25$.