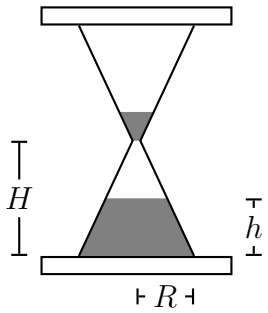
Douglass Houghton Workshop, Section 1, Mon 12/08/25

Worksheet Where the Wild Things Are

1. The lower chamber of an hourglass is shaped like a cone with height H inches and base radius R inches, as shown in the figure to the right, above. Sand falls into this cone. Write an expression for the volume of the sand in the lower chamber when the height of the sand there is h in (Hint: A cone with base radius r and height y has volume $V = \frac{1}{3}\pi r^2 y$, and it may be helpful to think of a difference between two conical vol umes.) Then, if $R = 0.9$ in, $H = 2.7$ in, and sand is falling into the lower chamber at $2\,\text{in}^3/\text{min}$, how fast is the height of the sand in the lower chamber changing when $h = 1$ in?



2. (Fall, 2011) For positive A and B, the force between two atoms is a function of the distance, r, between them:
$$f(r) = -\frac{A}{r^2} + \frac{B}{r^3}.$$
   (a) Find the zeroes of f in terms of A and B.
   (b) Find all critical points and inflection points of f in terms of A and B.
   (c) If f has a local minimum at $(1, -2)$ find the values of A and B. Using your values for A and B, justify that $(1, -2)$ is a local minimum.

3. (Winter, 2010) Suppose that the standard price of a round-trip plane ticket from Detroit to Paris, purchased t days after April 30, is $P(t)$ dollars. Assume that P is an invertible function (even though this is not always the case in real life). In the context of this problem, give a practical interpretation for each of the following:
   (a) $P'(2) = 55$
   (b) $\displaystyle\int_5^{10} P'(t)\,dt$
   (c) $P^{-1}(690)$
   (d) $\displaystyle\frac{1}{5}\int_5^{10} P(t)\,dt$

4. Hanna is studying a colony of *campylobacter jejuni* bacteria. She finds that the growth rate of the colony is increasing exponentially. That is, if $P(t)$ is the population in thousands after t hours, then $P'(t) = Ae^{kt}$ for some constants A and k.
   (a) Suppose there are 1000 bacteria at the start of the experiment. Write an integral which gives the number of bacteria present after T hours.
   (b) Use the Fundamental Theorem of Calculus to get a formula without an integral for the number of bacteria after T hours.
   (c) Suppose the bacteria grew at an initial rate of 500 bacteria per hour, and after 6 hours the rate has increased to 1000 bacteria per hour. Find values for the constants A and k.
   (d) How many bacteria are there 6 hours after the experiment started?

5. Brahm also does an experiment with the same starting population of bacteria, but he plays cello music to the bacteria. He finds the bacteria LOVE Jazz, and they grow while a song is playing and stop growing between songs. Brahm plays them a series of 3-minute songs with 3-minute breaks between them, and finds that t hours after the experiment starts, the growth rate (in thousands per hour) is $1+\sin(20\pi t)$. How many bacteria grow in each 6-minute cycle?

6. In Summer 2018, it was reported that ocean levels were rising at the rate of $\frac{1}{2}$ mm per year, and that that rate had tripled over the previous decade. If that tripling continues, how long will it be until the sea level has risen half a meter? 1 meter?

7. (Fall, 2016) Isabelle has a small orchard where she grows Michigan apples. After careful study last season, Isabelle found that the total cost, in dollars, of producing a bushels of apples can be modeled by

$$C(a) = -25500 + 26000e^{0.002a}$$

for $0 \leq a \leq 320$. Morgan has promised to buy up to 100 bushels of apples for her famous apple ice cream. If Isabelle has any remaining apples, she has an agreement to sell them to Uday's cider mill at a reduced price. Let $R(a)$ be the revenue generated from selling a bushels of apples. Then

$$R(a) = \begin{cases} 70a & \text{if } 0 \leq a \leq 100 \\ 2000 + 50a & \text{if } 100 < a \leq 320 \end{cases}$$

   (a) How much will Uday's cider mill pay per bushel?

   (b) What is Isabelle's fixed cost?

   (c) For what values of a will Isabelle's marginal revenue equal her marginal cost?

   (d) Graph marginal revenue and marginal cost.

   (e) Assuming Isabelle can produce up to 320 bushels of apples, how many bushels should she produce, and what will be the maximum profit?

8. (This problem appeared on the Fall, 2008 Math 115 Final Exam) Suppose that you are brewing coffee and that hot water is passing through a special, cone-shaped filter. Assume that the height of the conic filter is 3 in. and that the radius of the base of the cone is 2 in. If the water is flowing out of the bottom of the filter at a rate of 1.5 in^3/min when the remaining water in the filter is 2 in. deep, how fast is the depth of the water changing at that instant?