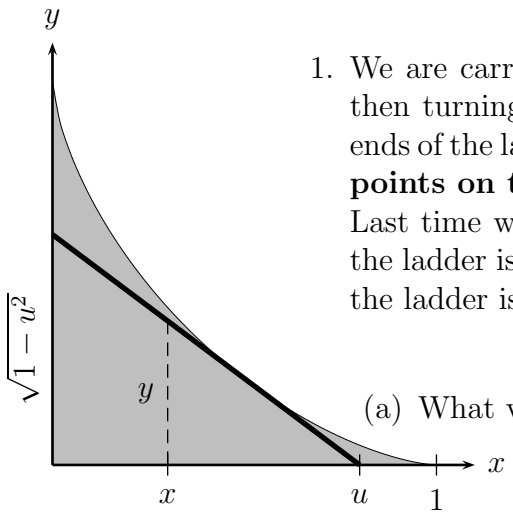
Douglass Houghton Workshop, Section 1, Mon 12/01/25

Worksheet Unleash the Dogs of War

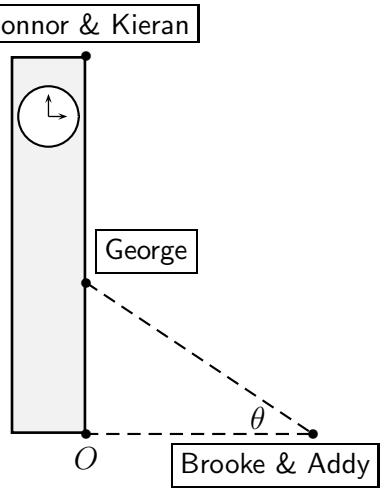


1. We are carrying a ladder of length 1 down a hallway, and then turning it to get around a corner, always keeping the ends of the ladder against the walls. The question is: **Which points on the floor does the ladder pass over?**
   Last time we found that if $0 \le x \le u \le 1$ and the base of the ladder is at $(u, 0)$, then the distance from $(x, 0)$ north to the ladder is
   $$y = \frac{u - x}{u}\sqrt{1 - u^2}.$$
   (a) What value of u maximizes y? (Keep x fixed!)

   (b) So for a fixed x, what is the maximum value of y, as the ladder moves?

   (c) You have found a formula for the curve at the top of the region we want. Simplify until it's beautiful. (This is the best part, so don't stop until it's truly wondrous.)

2. (Adapted from a Winter, 2005 Math 115 exam) One day Connor & Kieran notice that the door to the Burton Tower carillon has been left open. They can't resist the urge to climb to the top of the tower and barricade themselves in. Connor then launches in to a lecture on quantum physics, accompanied by Kieran on the trumpet.
   At the University's behest, George calls on his ROTC training and begins climbing the outside of the tower in order to liberate it. Meanwhile on the ground, 30 feet from the tower, Brooke and Addy give an impromptu rendition of Smashing Pumpkins songs on dueling guitars. They look up at an angle θ to see George.



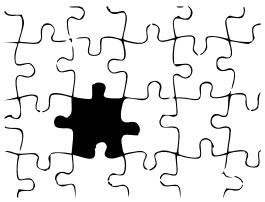
   (a) Find the rate of change of George's distance from the point O with respect to θ.

   (b) If the distance from point O to Connor & Kieran is 200 ft and George climbs at a constant 8 ft/sec, what is the rate of change of θ with respect to time when George is halfway up?

   (c) When George is halfway up, Connor & Kieran drop the end of a a rope down to help him. The end of the rope falls with a constant acceleration of 32 ft/sec^2. When does George catch it, and what is its speed when he does?

   (d) Brooke & Addy watch the end of the rope as it drops, and also begin backing away from the tower at a rate of 5 ft/sec. How fast is the angle of their gaze changing when George catches the rope?

3. (Fall, 2014) Ava and Michelle (who is a puzzle expert) start a business manufacturing and selling jigsaw puzzles made from Ava's photographs of National Parks. (Crater Lake is their biggest seller.) After careful study, Ava and Michelle have determined that they can produce up to 160 puzzles in a year. They can sell the first 100 puzzles to book stores and any remaining puzzles to wholesalers. The revenue in dollars from selling x puzzles will be



$$R(x) = \begin{cases} 6x & \text{if } 0 \le x \le 100 \\ 4x + 200 & \text{if } 100 < x \le 160. \end{cases}$$

(a) What is the price book stores pay for each puzzle?

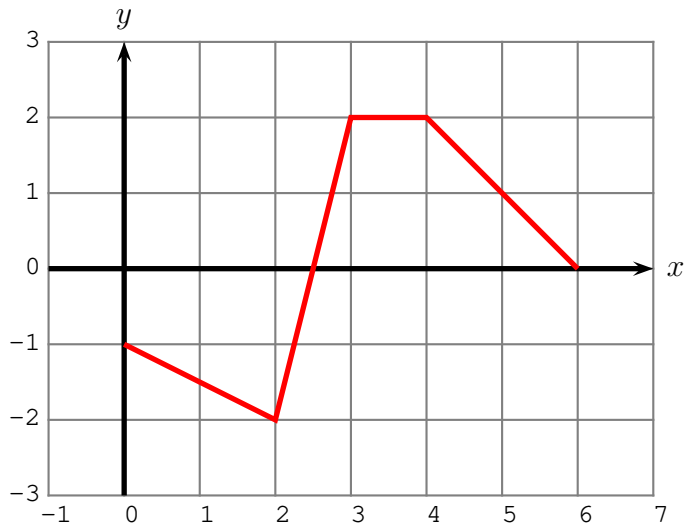
(b) What is the price wholesalers pay?

(c) It costs $C(x) = 20 + 3x + 24\sqrt{x}$ to produce x puzzles. (Use that formula for the rest of the problem.) What is the fixed cost of Ava and Michelle's operation?

(d) At what production levels does marginal revenue equal marginal cost?

(e) How many puzzles should Ava and Michelle produce to maximize their profit, and what is the maximum possible profit?

4. (Winter, 2024) An **even** function $g(x)$, which is defined for all real numbers, is graphed on the interval $[0, 6]$ below. Note that $g(x)$ is piecewise linear.



(a) The function $g(x)$ has a continuous antiderivative, $G(x)$, which satisfies $G(2) = -2$. Complete the following table of values for $G(x)$.

x	-6	-4	-2	0	2	4	6
$G(x)$					-2		

(b) Sketch a graph of $G(x)$ on the interval $[-6, 6]$. Make sure to clearly label the values at the points in your table above and also make it clear where $G(x)$ is increasing or decreasing, and where $G(x)$ is concave up, concave down, or linear.