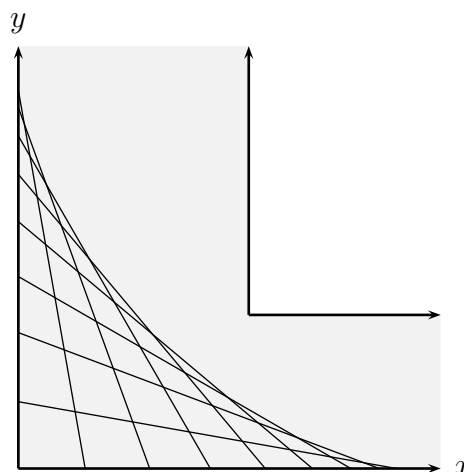


## Worksheet To Infinity, and Beyond

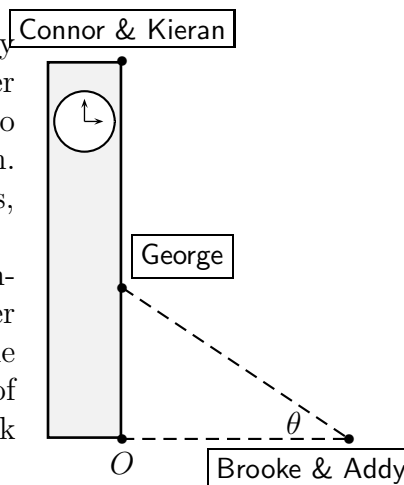
1. Suppose we are carrying a ladder down a hallway, and then turning it to get around a corner, always keeping the ends of the ladder against the walls. The question is: **Which points on the floor does the ladder pass over?** Let's assume the ladder has length 1. In the picture, the gray area is the hallway and the fine lines are the ladder in different positions.



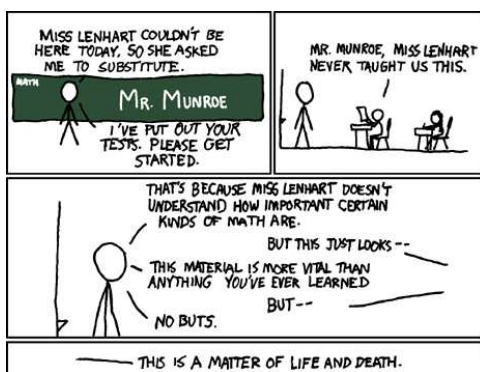
- (a) Suppose the base of the ladder is at the point  $(u, 0)$ . Where on the  $y$ -axis is the top of the ladder? Draw a picture!
- (b) Suppose you are standing at  $(x, 0)$  and looking north (up the page). If  $x < u$ , how far away do you see the ladder?

To be continued...

2. (Adapted from a Winter, 2005 Math 115 exam) One day Connor & Kieran notice that the door to the Burton Tower carillon has been left open. They can't resist the urge to climb to the top of the tower and barricade themselves in. Connor then launches in to a lecture on quantum physics, accompanied by Kieran on the trumpet. At the University's behest, George calls on his ROTC training and begins climbing the outside of the tower in order to liberate it. Meanwhile on the ground, 30 feet from the tower, Brooke and Addy give an impromptu rendition of Smashing Pumpkins songs on dueling guitars. They look up at an angle  $\theta$  to see George.



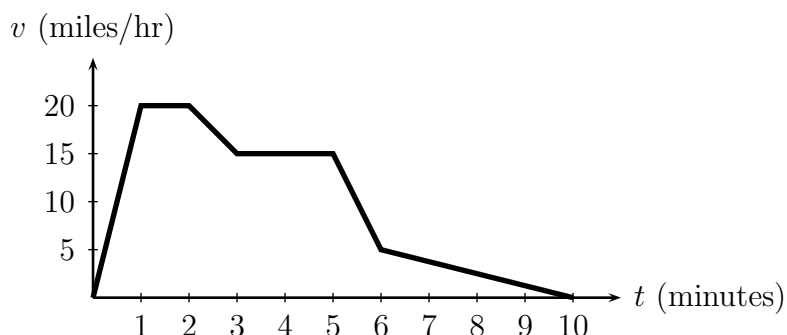
- (a) Find the rate of change of George's distance from the point  $O$  with respect to  $\theta$ .
- (b) If the distance from point  $O$  to Connor & Kieran is 200 ft and George climbs at a constant 8 ft/sec, what is the rate of change of  $\theta$  with respect to time when George is halfway up?
- (c) When George is halfway up, Connor & Kieran drop the end of a rope down to help him. The end of the rope falls with a constant acceleration of 32 ft/sec<sup>2</sup>. When does George catch it, and what is its speed when he does?
- (d) Brooke & Addy watch the end of the rope as it drops, and also begin backing away from the tower at a rate of 5 ft/sec. How fast is the angle of their gaze changing when George catches the rope?



The velociraptor spots you 40 meters away and attacks, accelerating at  $4 \text{ m/s}^2$  up to its top speed of  $25 \text{ m/s}$ . When it spots you, you begin to flee, quickly reaching your top speed of  $6 \text{ m/s}$ . How far can you get before you're caught and devoured?



- 3.
4. (This problem appeared on the Winter, 2005 Math 115 Final Exam) In a certain episode of The Simpsons, Homer needs to deliver Lisa's homework to her at school, and he must do so before Principal Skinner arrives. Suppose Homer starts from the Simpson home in his car and travels with velocity given by the figure below. Suppose that Principal Skinner passes the Simpson home on his bicycle 2 minutes after Homer has left, following him to the school. Principal Skinner is able to sail through all the traffic and travels with constant velocity 10 miles per hour.



- (a) How far does Homer travel during the 10 minutes shown in the graph?
- (b) What is the average of Homer's velocity during the 10 minute drive?
- (c) At what time,  $t > 0$ , is Homer the greatest distance ahead of Principal Skinner?
- (d) Does Principal Skinner overtake Homer, and if so, when? Explain.
- (e) If Skinner continues at 10 mph, and if Homer's velocity continues to decrease at a constant rate, then how long does it take for Homer and Skinner to meet?
5. (Fall, 2011) For positive  $A$  and  $B$ , the force between two atoms is a function of the distance,  $r$ , between them:

$$f(r) = -\frac{A}{r^2} + \frac{B}{r^3}.$$

- (a) Find the zeroes of  $f$  in terms of  $A$  and  $B$ .
- (b) Find all critical points and inflection points of  $f$  in terms of  $A$  and  $B$ .
- (c) If  $f$  has a local minimum at  $(1, -2)$  find the values of  $A$  and  $B$ . Using your values for  $A$  and  $B$ , justify that  $(1, -2)$  is a local minimum.