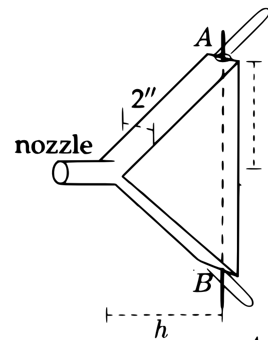
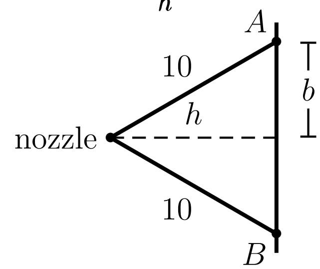
Douglass Houghton Workshop, Section 1, Mon 11/17/25

Worksheet Sweet Sorrow





1. (This problem appeared on a Winter, 2008 Math 115 exam) A bellows has a triangluar frame made of three rigid pieces. Two pieces, each 10 inches long, are hinged at the nozzle. They are attached to the third piece at points A and B which can slide, as shown in the diagrams below. (The figures show a 3D sketch of the bellows and a 2D sketch that may be specifically useful to solve the problem.)
   Each piece of the frame is 2 inches wide, so the volume (in cubic inches) of air inside the bellows is equal to the area (in square inches) of the triangluar cross-section above, times 2. Suppose you pump the bellows by moving A downward toward the center at a constant speed of 3 in/sec. (So B moves upwards at the same speed.) What is the rate at which air is being pumped out when A and B are 12 inches apart? (So A is 6 inches from the center of the vertical piece of the frame.)

2. (Adapted from a Fall, 2004 Math 115 final) Kieran's family spend Thanksgiving eating scones. The rate at which they eat scones is given by the function $r(t)$, where t is measured in hours and $r(t)$ is in dozens of scones/hour. Suppose $t = 0$ corresponds to 10 am, when the binge begins.

   (a) Write a definite integral that represents the total amount of scones Kieran's family consume between noon and 10 pm.

   (b) If Kieran's family's rate of eating scones is given by $r(t) = 10e^{-t} + 1$, use a left hand sum with three (3) subdivisions to estimate the amount of scones Kieran's family eat in the first four hours of their binge.

   (c) Should your estimate in part (b) be an underestimate or an overestimate? Explain.

3. (From the Winter, 2007 Math 115 Final Exam) Suppose that f and g are continuous functions with

$$\int_0^2 f(x)\,dx = 5 \qquad \text{and} \qquad \int_0^2 g(x)\,dx = 13.$$

   Compute the following. If the computation cannot be made because something is missing, explain clearly what is missing.

   (a) $\displaystyle\int_4^6 f(x-4)\,dx$

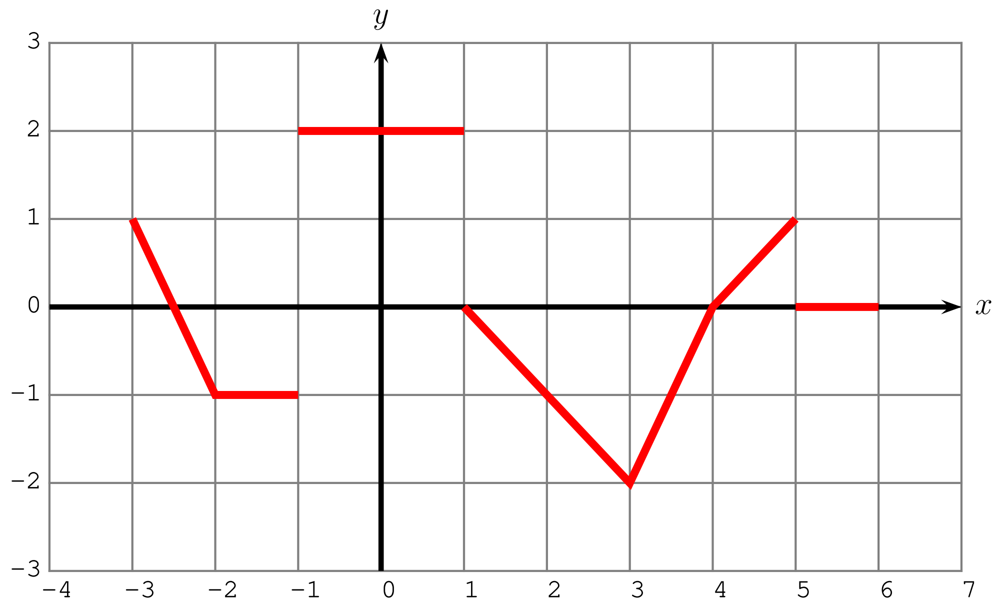
   (b) $\displaystyle\int_{-2}^0 2g(-t)\,dt$

   (c) $\displaystyle\int_2^0 (f(y)+2)\,dy$

   (d) $\displaystyle\int_2^2 g(x)\,dx$

   (e) Suppose that f is an even function. Find the average value of f from -2 to 2.

4. (Fall, 2018) Part of the graph of a piecewise-linear function $r(x)$ is shown below.



The function $h(x)$ is a continuous antiderivative of $r(x)$ with $h(0) = 1$. Sketch the graph of $h(x)$ over the interval $-3 \leq x \leq 6$. Make sure to pay attention to:

- where h is and is not differentiable.
- where h is increasing/decreasing/constant.
- where h is linear/concave up/concave down.
- the values of $h(-3), h(-2), h(-1), \ldots, h(5), h(6)$.

5. Suppose $\int_4^9 (4f(x) + 7)\, dx = 315$. Find $\int_4^9 f(x)\, dx$.

6. (Winter, 2010) Suppose that the standard price of a round-trip plane ticket from Detroit to Paris, purchased t days after April 30, is $P(t)$ dollars. Assume that P is an invertible function (even though this is not always the case in real life). In the context of this problem, give a practical interpretation for each of the following:

   (a) $P'(2) = 55$
   (b) $\int_5^{10} P'(t)\, dt$
   (c) $P^{-1}(690)$
   (d) $\frac{1}{5}\int_5^{10} P(t)\, dt$

7. (Fall, 2011) For positive A and B, the force between two atoms is a function of the distance, r, between them:

$$f(r) = -\frac{A}{r^2} + \frac{B}{r^3}.$$

   (a) Find the zeroes of f in terms of A and B.
   (b) Find all critical points and inflection points of f in terms of A and B.
   (c) If f has a local minimum at $(1, -2)$ find the values of A and B. Using your values for A and B, justify that $(1, -2)$ is a local minimum.