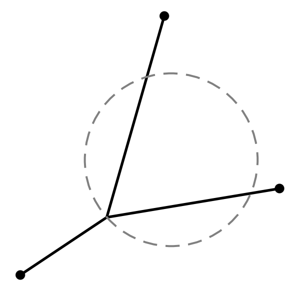
Douglass Houghton Workshop, Section 1, Wed 11/12/25

Worksheet Remember What Peace There May Be in Silence

1. Shortest Network. Last time we used calculus to show that a Λ-shaped network can be improved if the vertex angle is less than 120°.

   (a) Prove that the network to the right is NOT minimizing. You don't need to find the optimal network, just prove that this one can be improved. Hint: consider the portion of the network that is inside the circle.

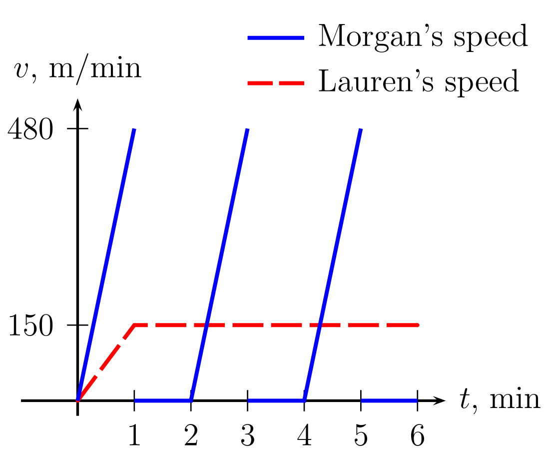
   

   (b) What allowed that trick to work? Phrase your answer like this: "Any network which contains ______ can be improved."

   (c) Put it all together, and explain where the soap puts the roundabout.

2. (Adapted from a Winter 2009 Math 115 Exam.) Lauren and Morgan, after much friendly trash talk about who's the fastest runner, decide to have a race. The two employ very different approaches.

   - Lauren takes the first minute to accelerate to a slow and steady pace which she maintains through the remainder of the race.
   - Morgan, on the other hand, spends the first minute accelerating to faster and faster speeds until she's exhausted and has to stop and rest for a minute—and then she repeats this process until the race is over. The graph below shows their speeds (in meters per minute), t minutes into the race. (Assume that the pattern shown continues for the duration of the race.)

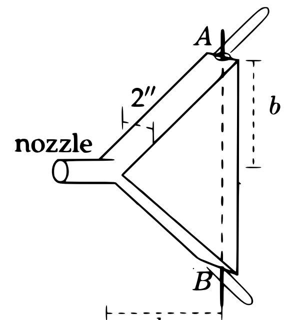
   (a) What is Lauren's average speed over the first two minutes of the race? What is Morgan's?

   (b) Morgan immediately gets ahead of Lauren at the start of the race. How many minutes into the race does Lauren catch up to Morgan for the first time?

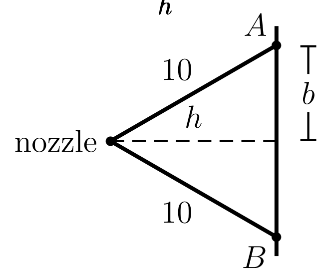
   (c) Draw graphs of Morgan's and Lauren's positions at time t. Be as precise as possible.

   (d) If the race is 720 meters total, who wins? What if it's 721 meters?

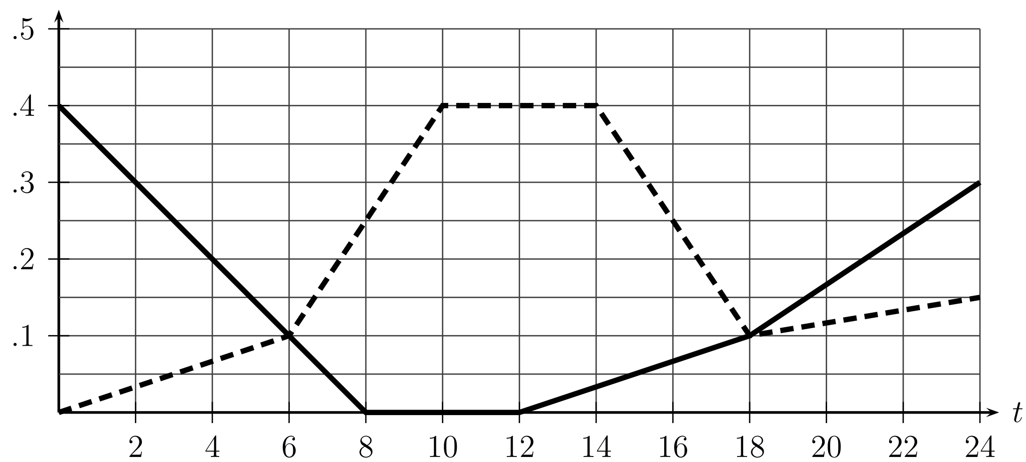
3. (This problem appeared on a Winter, 2008 Math 115 exam) A bellows has a triangluar frame made of three rigid pieces. Two pieces, each 10 inches long, are hinged at the nozzle. They are attached to the third piece at points A and B which can slide, as shown in the diagrams below. (The figures show a 3D sketch of the bellows and a 2D sketch that may be specifically useful to solve the problem.)



Each piece of the frame is 2 inches wide, so the volume (in cubic inches) of air inside the bellows is equal to the area (in square inches) of the triangluar cross-section above, times 2. Suppose you pump the bellows by moving A downward toward the center at a constant speed of 3 in/sec. (So B moves upwards at the same speed.) What is the rate at which air is being pumped out when A and B are 12 inches apart? (So A is 6 inches from the center of the vertical piece of the frame.)



4. (Fall, 2011) Suppose the graph below shows the rate of snow melt and snowfall on Mount Arvon, the highest peak in Michigan (at a towering 1970 ft), during a day (24 hour period) in April of last year. The function $m(t)$ (the solid curve) is the rate of snow melt, in inches per hour, t hours after the beginning of the day. The function $p(t)$ (the dashed curve) is the snowfall rate in inches per hour t hours after beginning of the day. There were 18 inches of snow on the ground at the beginning of the day.



(a) Over what period(s) was the snowfall rate greater than the snow melt rate?

(b) When was the amount of snow on Mount Arvon increasing the fastest? When was it decreasing the fastest?

(c) When was the amount of snow on Mount Arvon the greatest? Explain.

(d) How much snow was there on Mount Arvon at the end of the day (at $t = 24$)?

(e) Sketch a well-labeled graph of $P(t)$, an antiderivative of $p(t)$ satisfying $P(0) = 0$. Label and give the coordinates of the points on the graph of $P(t)$ at $t = 10$ and $t = 18$.