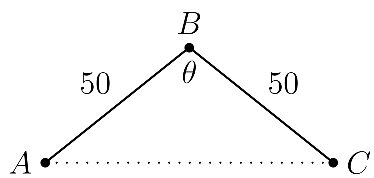
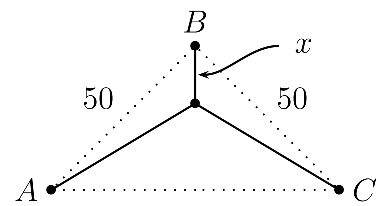
Douglass Houghton Workshop, Section 1, Mon 11/10/25

Worksheet Question Everything

1. Shortest Network. So far we've looked at the case where the cities are at the corners of an isosceles triangle like the one below. We know:



- When angle B is $70°$ or $90°$, it is possible to improve upon the Λ-shaped network shown by building a roundabout south of B and connecting it to all three cities.
- However, when B is $150°$, the Λ is better than all possible ⅄'s.



   (a) Suppose the measure of angle B is θ. Use the law of cosines to write a formula for the length of the ⅄-shaped network to the right, in terms of θ and x.

   (b) Call that function $L_\theta(x)$. Put your calculator in degrees mode and plot $L_{70}(x)$, $L_{90}(x)$, and $L_{150}(x)$, for x from 0 to 50. Put the graphs on the board.

   (c) The shapes seem about the same. What can you see in the graphs that explains the fact that in two cases the Λ can be improved, and in the other it can't? (Remember the Λ is $x = 0$.)

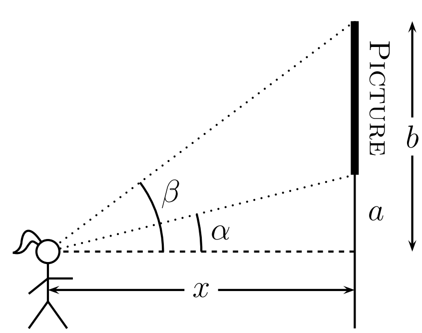
   (d) Use calculus to figure out which Λ's can be improved, and which can't. State the result in the form: "Any Λ-shaped network with an angle smaller than ______ can be improved".

   (e) This is for those who like to compute and simplify. Show that the function $L_\theta(x)$ defined above is always concave up, by finding and simplifying its second derivative.

2. Michelle has created a masterwork of cross-stitch—a huge representation of Michigan Stadium. She took a detailed picture of it hopes to someday turn it into a jigsaw puzzle. For now, it is mounted on the wall. Its bottom is a feet above eye level, and its top is b feet above eye level. If you stand far away from the wall, you can't see the picture well. But if you stand close to the wall, you can't see well either! So the question is: how far from the wall should you stand in order to have the best view?

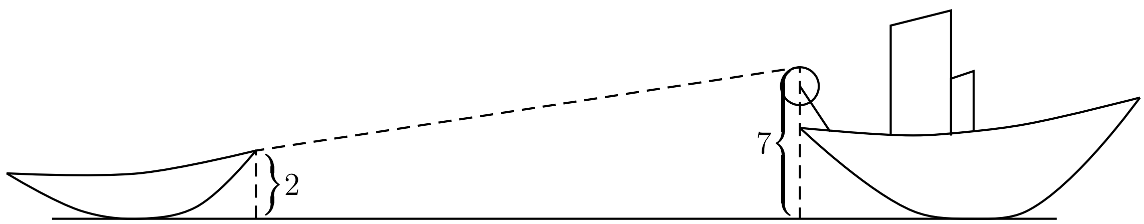
   (a) Let α and β be the angles between eye level and the bottom and top of the picture, as shown. x is your distance from the wall. Find α and β in terms of x, a, and b.

   (b) $\beta - \alpha$ is the angle that the picture takes up in your field of vision. So find the value of x that maximizes $\beta - \alpha$.



3. (Adapted from a Fall, 2006 Math 115 Final Exam.) Elijah is sailing his Flying Scot dinghy on Lake Michigan with his Dad. Unfortunately the wind has died around sunset, and it's important that they get home in time for dinner, so Elijah hails a passing tugboat to give the boat a tow.

   The tugboat captain throws Elijah a line, and he attaches it to his boat 2 meters above the water line. The other end of the cable is attached to a wheel of radius 0.5 meters sitting on the back of the tugboat. The top of the wheel is 7 meters above the water, and turns at a constant rate of 1 revolution per second. [See the figure below—not drawn to scale.]



   (a) At what rate is the length of the cable between the boat and the tugboat changing?

   (b) How fast is Elijah's boat being pulled forward when it is 10 meters away from the tugboat?

4. November is National Novel Writing Month, and many people around the country attempt to complete a first draft of a novel in the course of the month. One of them is Mark's friend Chris. At the end of every day November she uploaded her manuscript to a website (`nanowrimo.org`), which counts how many words she has written. Here are her counts, rounded to the nearest hundred:

Nov.	Count	Nov.	Count	Nov.	Count	Nov.	Count	Nov.	Count
1	1700	5	8300	9	9400	13	14800	17	23300
2	3600	6	8300	10	9400	14	17000	18	24700
3	5800	7	8700	11	11800	15	20100	19	26600
4	7500	8	8700	12	13800	16	21300	20	26700

   (a) Let x be the time in days since the start of November, and let $W(x)$ be the total number of words Chris has written at time x. Assume that each day Chris writes at a steady rate, from midnight to midnight. (But different rates for different days.) Draw a graph of $W(x)$ for the second week (x from 7 to 14).

   (b) Let $w(x)$ be the derivative of $W(x)$. Draw a graph of $w(x)$ for x from 7 to 14.

   (c) Now consider the function $F(t)$, which is the area between the line $x = 7$, the line $x = t$, the x-axis, and the graph of $w(x)$. Make a table of values showing $F(7), F(8), \ldots, F(14)$. What do you notice? Explain this result.