Douglass Houghton Workshop, Section 1, Mon 10/27/25

Worksheet Now is the Winter of our Discontent

1. Last time we investigated rules for how a population of manatees might change. Let's nail down the essential features of all similar rules. Here's what we know:

Rule	Equilibrium	Stable?
$P(n+1) = 1.5P(n) - 200$	400	No
$P(n+1) = .75P(n) + 125$	500	Yes

An **equilibrium** is a population that will stay constant from year to year. An equilibrium $\hat{P}$ is **stable** if when the population starts a little above or below $\hat{P}$, it moves toward $\hat{P}$. Otherwise $\hat{P}$ is **unstable**.

(a) Add rows to the table for these rules. You can reason either numerically, graphically, algebraically, or with words.

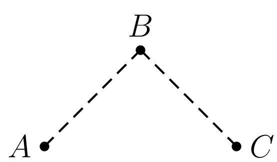
$P(n+1) = .4P(n) + 600$ $\quad$ $P(n+1) = -1.3P(n) + 460$

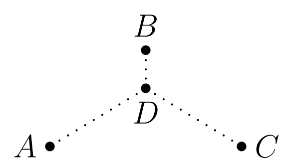
$P(n+1) = 1.1P(n) - 330$ $\quad$ $P(n+1) = P(n) + 300$

$P(n+1) = -.5P(n) + 1200$ $\quad$ $P(n+1) = -P(n) + 300$

(b) Now do $P(n+1) = mP(n) + b$, where m and b are constants.

2. The three cities in the pictures below are at the corners of a 45°-45°-90° triangle whose legs are 50 miles long. The three mayors, working together, would like to build roads between them in such a way that there is a way to get from any one city to any other city.





(Say, A is Ann Arbor, B is Flint, and C is Port Huron.) The first, simple proposal (on the left) is to build a road from A to B and another from B to C. That would certainly work. But roads are expensive, and one of the mayors (who, luckily, studied calculus) proposes building roads from A and C to a point D just south of B, then building a road north from there to B.

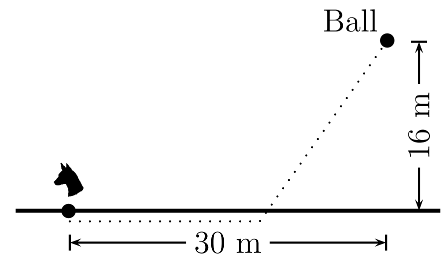
(a) Let x be the length of the north-south road in the second proposal. What does it mean if $x = 0$?

(b) Calculate the total length of the new network in terms of x. Hint: "Law of cosines".

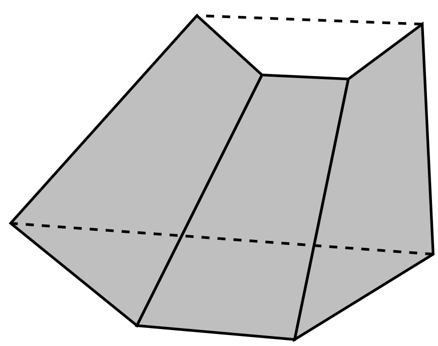
(c) Can you find a value of x which will produce a shorter network than the simple proposal?

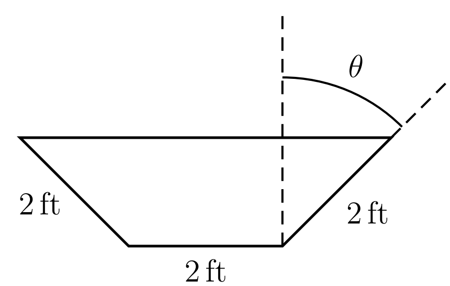
3. Suppose Gianna is walking along the shore of Kent Lake in Island State Park, with her dog Bentley. Gianna throws a ball 30 meters down the beach and 16 meters out into the water.

Bentley, being practical, wants to get to the ball as quickly as possible. The thing is that he can run faster than he can swim; his running speed on the beach is 9 meters per second, and he can swim 3 meters per second. How should Bentley (who has an intuitive notion of calculus) get to the ball?



4. A trough, as shown below, is to be made with a base that is 2 feet wide and 10 feet long. The sides of the trough are also 2 feet wide by 10 feet long, and are to be placed so they make an angle θ with the vertical.





(a) What is the area, in terms of θ, of a cross section of the trough perpendicular to its long side? What is the volume of the trough?

(b) What angle θ will give the trough the largest volume, and what is that volume? [Hint: you can always replace $\cos^2(\theta)$ with $1 - \sin^2(\theta)$.]

5. (This problem appeared on a Winter 2007 Math 115 exam) Suppose f and g are differentiable functions with values given by the table below.

(a) If $h(x) = f(x)g(x)$, find $h'(3)$.

(b) If $j(x) = \dfrac{(g(x))^3}{f(x)}$, find $j'(1)$.

(c) If $d(x) = x \ln\left(e^{f(x)}\right)$, find $d'(3)$.

(d) If $t(x) = \cos(g(x))$, find $t'(1)$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	2	9	-3	7
3	4	11	15	-19