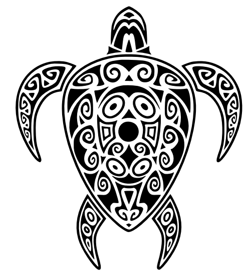
Douglass Houghton Workshop, Section 1, Wed 10/22/25

Worksheet May the Road Rise to Meet You



1. Isabelle and Chris are studying a population of turtles in California. Suppose that the population changes according to the rule:

$$P(n+1) = 1.5P(n) - 200$$

where $P(0)$ is the population in 2025, $P(1)$ is the population 1 year later, etc. (P is measured in turtles.)

   (a) Make up a (short) story about turtles that explains the formula above.

   (b) Suppose $P = 320$ in 2025. What will happen in the long run?

   (c) Suppose instead that $P = 440$ in 2025. Now what happens?

   (d) A population is in **equilibrium** if it stays the same from year to year. Is there an equilibrium number for this population?

   (e) Explain these results pictorially by drawing the graphs of $y = x$ and $y = 1.5x - 200$. Start at $(320, 320)$, go down to the other graph, and then over to $y = x$. That's the new population. Repeat. Then start at 440.

2. Repeat the last problem, but for the rule

$$P(n+1) = .75P(n) + 125.$$

3. (From Fall, 2011, Math 115 Exam 2) Let $f(x) = \ln(x)$. Use the table of values below for $g(x)$ and $g'(x)$ to answer the following questions.

   (a) If $F(x) = f(g(x))$, find $F'(4)$.

   (b) If $G(x) = g^{-1}(x)$, find $G'(4)$.

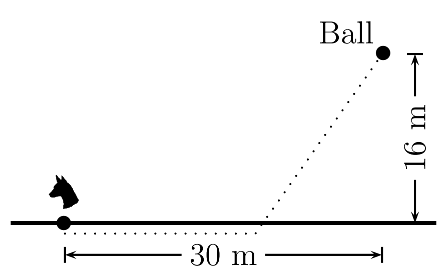
   (c) If $H(x) = \tan(g(x))$, find $H'(3)$.

   (d) If $E(x) = e^{f(x)g(x)}$, find $E'(2)$.

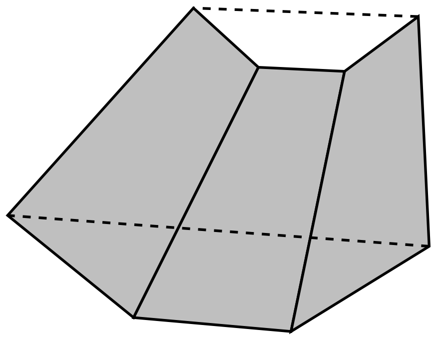
x	2	3	4
$g(x)$	1	4	6
$g'(x)$	5	3	2

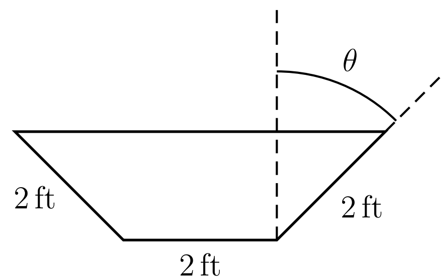
4. Suppose Gianna is walking along the shore of Thompson Lake, near Howell, with her dog Bentley. Gianna throws a ball 30 meters down the beach and 16 meters out into the water.

Bentley, being practical, wants to get to the ball as quickly as possible. The thing is that he can run faster than he can swim; his running speed on the beach is 9 meters per second, and he can swim 3 meters per second. How should Bentley (who has an intuitive notion of calculus) get to the ball?



5. A trough, as shown below, is to be made with a base that is 2 feet wide and 10 feet long. The sides of the trough are also 2 feet wide by 10 feet long, and are to be placed so they make an angle θ with the vertical.





(a) What is the area, in terms of θ, of a cross section of the trough perpendicular to its long side? What is the volume of the trough?

(b) What angle θ will give the trough the largest volume, and what is that volume? [Hint: you can always replace $\cos^2(\theta)$ with $1 - \sin^2(\theta)$.]

6. (This problem appeared on a Winter 2007 Math 115 exam) Suppose f and g are differentiable functions with values given by the table below.

(a) If $h(x) = f(x)g(x)$, find $h'(3)$.

(b) If $j(x) = \dfrac{(g(x))^3}{f(x)}$, find $j'(1)$.

(c) If $d(x) = x \ln\left(e^{f(x)}\right)$, find $d'(3)$.

(d) If $t(x) = \cos(g(x))$, find $t'(1)$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	2	9	-3	7
3	4	11	15	-19