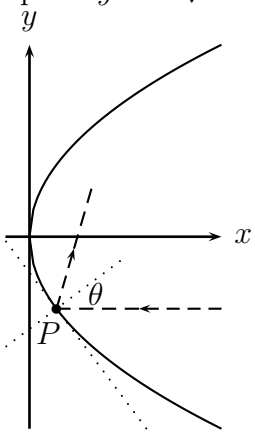
Douglass Houghton Workshop, Section 1, Mon 10/06/25

Worksheet Kangaroo

1. Last time we thought about a parabolic mirror in the shape of the graph of $y = \pm\sqrt{4x}$.

   So far we've found:

   - A light ray $y = -b$ hits the mirror at $P = (b^2/4, -b)$.
   - The slope of the tangent at that point is $-2/b$.
   - The normal line at the same point has slope $b/2$.
   - When a line makes an angle θ with the x-axis, it has slope $\tan\theta$.
   - So if we call the angle between the normal line and the horizontal θ, then $\theta = \tan^{-1}(b/2)$.



   (a) Draw the picture on the board.

   (b) To the ray, the mirror looks flat, just like the tangent line. Draw the reflected ray. What angle does it make with the x-axis?

   (c) We know that $\sin 2x = 2\sin x\cos x$ and $\cos 2x = \cos^2 x - \sin^2 x$. Use those to find a formula for $\tan 2x$ in terms of $\tan x$.

   (d) What is the slope of the reflected ray?

   (e) Write an equation for the reflected ray.

   (f) Where does the reflected ray intersect the x-axis? What is surprising about this answer?

   (g) Graph several rays, with their reflections.

   (h) What's cool about this type of mirror?

2. In "The 12 days of Christmas", a certain poultry-afficianado receives a number of gifts from her true love:

   Day 1: A partridge in a pear tree. How to get it down?

   Day 2: 2 turtle doves, and another partridge in a pear tree. Is it the same tree?

   Day 3: 3 French hens, 2 more turtle doves, and another partidge.

   ...

   Day 12: 12 drummers drumming (loudly), eleven pipers piping (make them stop!), ..., and yet another partridge in a pear tree.

   (a) How many total partridges does the heroine receive, over the course of the song? How many turtle doves?

   (b) If item 1 is "partridge", item 2 is "turtle dove", etc., then write a formula for the total number of item n's received.

   (c) Of which item does Mr. Truelove send the most? (Solve using calculus.)

3. (This problem appeared on the Fall, 2008 Math 115 Final Exam) At the Michigan-Ohio State basketball game this year, the Michigan Band discovers that the amount of time it spends playing "Hail to the Victors" has a direct impact on the number of points our team scores. If the band plays for x minutes, then the Wolverines will score

$$W(x) = -.48x^2 + 7.2x + 63$$

points. Assume that the band can play for a maximum of 10 minutes.

   (a) How long should the band play to maximize the number of points Michigan scores?

   (b) The band affects how many points Ohio State scores as well. x minutes of playing results in the Buckeyes scoring

$$B(x) = -x^2 + 8x + 84$$

   points. Find the number of minutes the band should play to maximize the margin of victory for Michigan.

   (c) What will be the score of the game for the case you found in part (b)?

4. Uday drives on the Pennsylvania Turnpike for a Go-Kart race in Philadelphia. He enters at the Ohio border, where a camera takes a picture of the car's license plate. The picture is processed and the licensce plate stored in a database, along with the time the picture was taken. Later Uday exits at Philadelphia, where another picture is taken. A week afterward, he receives a bill in the mail along with a speeding citation, stating that he was going exactly 75 mph at some point on his trip. How does the Mean Value Theorem allow the authorities to be sure that happened?

5. The diagrams below each have 4 regions, representing different ways a function can behave at a point. In each region write an example of a function and a point that meets the criteria. For example, in the intersection of "$f' = 0$" and "f changes direction", we have $x^2 \,@\, x = 0$, because the derivative of x^2 is indeed 0 at $x = 0$, and the function switches from decreasing to increasing there.