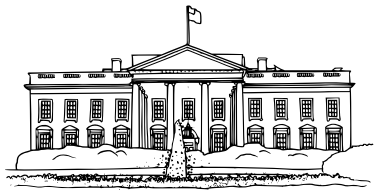
Douglass Houghton Workshop, Section 1, Mon 09/29/25

Worksheet Joy is not in things; it is in us



1. We still have this $1/z$ scale model of the White House, which we plan on blowing up. We want to decide what speed to run the film at, so that when we slow it down to 24 frames per second, we get a realistic explosion.

   (a) Last time we showed that an object will fall $16t^2$ feet in t seconds. So how long does it take for an object to fall off the real white house, which is H feet tall? How many frames will that be, if we film it at 24 frames per second and show it at the same speed?

   (b) How long does it take an object to fall off the top of the model?

   (c) How many frames per second should you film to get the right number of frames to make it look like the model is full-sized?

2. Consider a mirror in the shape of the graph of $y = \pm\sqrt{4x}$.

   (a) Draw the mirror (make it big). What shape is it?

   (b) Draw a light ray travelling leftward along the line $y = -b$, where b is some positive number (making $-b$ negative). At what point P does the ray hit the mirror?

   (c) Find, in terms of b, the slope of the tangent to the mirror at P.

   (d) The *normal* to a curve at a point is the line through that point which is perpendicular to the tangent line. Find the slope of the normal to the mirror at P, and draw both the normal and tangent lines on your graph.

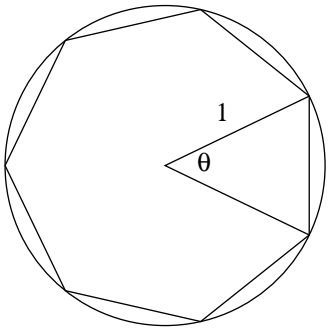
   (e) Suppose a line makes an angle θ with the positive x-axis. What is the slope of the line?

   (f) Let θ be the angle the normal to the mirror at P makes with the light ray $y = -b$. Can you write θ in terms of b? Hint: Use (2d) and (2e).

   To be continued...

3. (This problem appeared on a Winter, 2004 Math 115 exam. Really!) While exploring an exotic spring break location, you discover a colony of geese who lay golden eggs. You bring 20 geese back with you. Suppose each goose can lay 294 golden eggs per year. You decide maybe 20 geese isn't enough, so you consider getting some more of these magical creatures. However, for each extra goose you bring home there are less resources for all the geese. Therefore, for each new goose the amount of eggs produced will decrease by 7 eggs per goose per year. How many more geese should you bring back if you want to maximize the number of golden eggs per year laid?

4. (This problem explains why $\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1$, but only when θ is measured in radians.) Consider a regular n-sided polygon inscribed in a circle of radius 1.



(a) Let A_n be the area of the polygon. What does A_n approach as n gets large? $\lim_{n\to\infty} A_n =$ $\square$

(b) We can compute A_n by dividing the polygon up into triangles which have a vertex at the center. Let θ be the vertex angle (in radians). What is θ in terms of n?

(c) What happens to θ as n gets large?

(d) What is the area of one of the triangles, in terms of θ?

(e) What is A_n in terms of θ?

(f) Substitute into the equation from part (a) so that it includes θ's but not n's. Simplify it as much as you can. Hint: $\sin(2x)=2\sin(x)\cos(x)$.

(g) What would change if we measured θ in degrees instead of radians?

5. (An old team homework problem.) Let $f(x)=x^2-2x+13$ and $g(x)=-x^2-2x-5$.

(a) Draw $y=f(x)$ and $y=g(x)$ on the same set of axes. How many lines are tangent to both graphs?

(b) Find the equations of those lines.

6. (This problem appeared on a Fall, 2006 Math 115 exam) The Flux F, in millilitres per second, measures how fast blood flows along a blood vessel. Poiseuille's Law states that the flux is proportional to the fourth power of the radius, R, of the blood vessel, measured in millimeters. In other words $F=kR^4$ for some positive constant k.

(a) Find a linear approximation for F as a function of R near $R=0.5$. (Leave your answer in terms of k).

(b) A partially clogged artery can be expanded by an operation called an angioplasty, which widens the artery to increase the flow of blood. If the initial radius of the artery was 0.5 mm, use your approximation from part (a) to approximate the flux when the radius is increased by 0.1 mm.

(c) Is the answer you found in part (b) an under- or over-approximation? Justify your answer.