


Worksheet If Wishes Were Changes, We'd All Live in Roses

- Use the definition of the derivative to find $f'(x)$ when $f(x) = \sqrt{x}$. Hint: $(a-b)(a+b) = a^2 - b^2$.
- One beautiful day, Ava is hitting golf balls at Sleeping Bear Dunes in Northwest Michigan, when one ball, flying horizontally, goes over a cliff.
 - The ball's downward acceleration due to gravity is -32 ft/sec^2 . So what is its velocity t seconds after it is dropped? (Keep in mind that the derivative of velocity is acceleration, and the initial velocity is 0.)
 - How far has the ball fallen after t seconds?
 - Suppose the ball takes 3.5 seconds to hit the bottom. How high is the cliff?
 - Explain a general method for finding the height of any cliff.
- Suppose you construct a $1/z$ scale model of the White House, in order to film it blowing up. You will show the film at 24 frames per second. How many frames per second should you *film* so that when you slow the speed down, things will fall at believable speeds?
- Consider a mirror in the shape of the graph of $y = \pm\sqrt{4x}$.
 - Draw the mirror (make it big). What shape is it?
 - Draw a light ray travelling leftward along the line $y = -b$, where b is some positive number (making $-b$ negative). At what point P does the ray hit the mirror?
 - Find, in terms of b , the slope of the tangent to the mirror at P .
 - The *normal* to a curve at a point is the line through that point which is perpendicular to the tangent line. Find the slope of the normal to the mirror at P , and draw both the normal and tangent lines on your graph.
 - Suppose a line makes an angle θ with the positive x -axis. What is the slope of the line?
 - Let θ be the angle the normal to the mirror at P makes with the light ray $y = -b$. Can you write θ in terms of b ? Hint: Use (4d) and (4e).

To be continued...

5. Addy produces a range of kitchenware to honor her favorite comic book writers. Her new “Stan Lee” cups have been especially popular. Let $P(m)$ represent her profit, in thousands of dollars, if she produces m thousand cups, and let $F(m)$ represent the number of followers, in thousands, she will have on social media after she produces m thousand cups. Assume that both functions are invertible and differentiable.
- Write a complete sentence that gives a practical interpretation of the equation $F^{-1}(200) = 8$.
 - Write a mathematical equation using the functions P , F , and/or their inverses that represents the following statement:
If Addy makes a profit of 30 thousand dollars, she will have 250 thousand followers.
 - Complete the following sentence to give a practical interpretation of the equation $F'(10) = 32$:
If Addy makes 12 thousand cups instead of making 10 thousand cups, ...
 - Which one statement below that is best supported by the equation $(P^{-1})'(16) = 4$:
 - For every 4 thousand cups Addy produces, she makes an extra 16 thousand dollars in profit, roughly.
 - If Addy has made 15.5 thousand dollars in profit, and would like to make 500 more dollars in profit, she will need to produce about 2 thousand more cups.
 - If Addy goes from producing 16 thousand cups to producing 17 thousand cups then her profit will increase by about 4 thousand dollars.
 - If Addy produces an extra 500 cups after producing her first 4 thousand cups, then she will make about an extra 8 thousand dollars in profit.
6. (An old team homework problem.) Let $f(x) = x^2 - 2x + 13$ and $g(x) = -x^2 - 2x - 5$.
- Draw $y = f(x)$ and $y = g(x)$ on the same set of axes. How many lines are tangent to both graphs?
 - Find the equations of those lines.
7. (This is based on an old web homework problem) Consider the two functions $f(x) = \sin(x)$ and $g(x) = e^{k-x}$. We want to find the smallest positive k that makes the two graphs tangent.
- Draw both functions on the same set of axes, using $k = 1$.
 - Imagine you have a dial to turn, and the dial controls the value of k . When you increase k , how does the picture change? What we want to do is turn the dial to just the right place. Imagine what that would look like.
 - Let a be the x -value of the point of tangency. Write down two equations for what must be true at a if k is just right.
 - Solve for k and a .
 - Graph your two functions to see if they really are tangent.