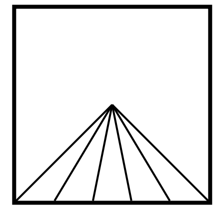
Douglass Houghton Workshop, Section 1, Wed 09/10/25

Worksheet Eat Cake (Let Them)

1. Cake! We had many successful ideas last time. One involved cutting a portion of the cake into triangles like the picture shown here. How do you cut the triangles so that the pieces have the same amount of cake and the same amount of frosting? Use that to find a solution for 12, 16, 20 people. Now find a solution for any multiple of 4. Now make that into a soultion for any number of people.



2. *Michael Phelps: The Sequel* Michael Phelps took all the money (let's say it's 2 million dollars) he got for endorsing Speedo, Visa, Subway, Frosted Flakes, and Head & Shoulders shampoo, and put it into a bank. The bank has several accounts available. For each, write an expression for how much Michael will have t years from now.

   (a) 6% interest, compounded annually.

   (b) 5% interest, compounded monthly.

   (c) 4% interest, compounded daily.

   (d) interest rate r, compounded n times per year.

   The bank also has something called "continuously compounded interest", which means that the number of compoundings per year is really really large. Write a limit expression for the amount of money he'll have if he gets interest rate r, compounded continuously.

3. Bankers and financial advisors use what they call the **Rule of 70**. It says:

   > If you invest money at annual interest rate r percent, it will take about $70/r$ years for your money to double.

   (So, for instance, \$500 invested at 5% interest will be worth \$1000 in about about 14 years, because $14 = 70/5$.)

   (a) Explain why the Rule of 70 works, and what assumptions you need to make it work. Hint: recall what we learned from Michael Phelps's towel:

   $$\lim_{n\to\infty}\left(1+\frac{T}{n}\right)^n = e^T.$$

   (b) Devise a similar rule for the time it takes your money to triple at r% interest.

4. (This problem appeared on a Fall, 2010 Math 115 exam.) Before the industrial era, the carbon dioxide (CO_2) level in the air in Ann Arbor was relatively stable with small seasonal fluctuations caused by plants absorbing CO_2 and producing oxygen in its place. Typically, on March 1, the CO_2 concentration reached a high of 270 parts per million (ppm), and on September 1, the concentration was at a low of 262 ppm. Let $G(t)$ be the CO_2 level t months after January 1.

   (a) Assuming that $G(t)$ is periodic and sinusoidal, sketch a neat, well-labeled graph of G with $t = 0$ corresponding to January 1.

   (b) Determine an explicit expression for G, corresponding to your sinusoidal graph above.

5. We've all seen 6-sided dice, and we presume they are "fair", in the sense that all 6 sides are equally likely to land on the bottom. Can you construct a fair 4-sided die? How about an 8-sided die? What other sizes are possible?

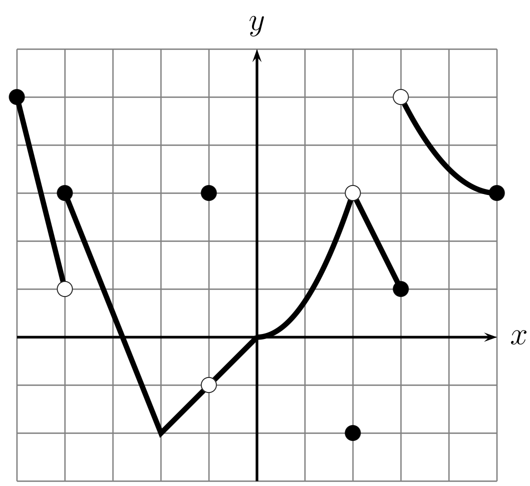
6. (This problem is adapted from a Fall, 2015 Math 115 Exam) Gianna is jumping rope while Morgan runs a stopwatch. There is a piece of tape around the middle of the rope. When the rope is at its lowest, the piece of tape is 2 inches above the ground, and when the rope is at its highest, the piece of tape is 68 inches above the ground. The rope makes two complete revolutions every second. When Morgan starts her stopwatch, the piece of tape is halfway between its highest and lowest points and moving downward. The height H (in inches above the ground) of the piece of tape can be modeled by a sinusoidal function $G(t)$, where t is the number of seconds displayed on Morgan's stopwatch.

   (a) Sketch a well-labeled graph of two periods of $G(t)$ beginning at $t = 0$.

   (b) Find a formula for $G(t)$.

   (c) Now Morgan takes a turn at jumping. Gianna resets the stopwatch and starts it over again. Let $M(w)$ be the height (in inches above the ground) of the piece of tape when Gianna's stopwatch says w seconds. A formula for $M(w)$ is $M(w) = 41 + 38\cos(2\pi w)$. Gianna is 60 inches tall. For how long (in seconds) during each revolution of the rope is the piece of tape higher than the top of Gianna's head? (Assume Gianna is standing straight while watching the stopwatch.)

7. (From a Fall, 2017 Math 115 Exam.) The graph of $y = Q(x)$ is shown. The gridlines are one unit apart.



   (a) On which of the following intervals is $Q(x)$ invertible?

   $[-4,-1]$ $[-2,3]$ $[2,5]$ $[-2,2]$

   (b) Find the following limits. Write "NI" if there you don't have enough information and "DNE" if the limit doesn't exist.

   i. $\lim_{x\to -1} Q(x)$ ii. $\lim_{w\to 2} Q(Q(w))$

   iii. $\lim_{h\to 0} \dfrac{Q(-3+h)-Q(-3)}{h}$ iv. $\lim_{x\to\infty} Q\left(\dfrac{1}{x}+3\right)$ v. $\lim_{x\to\frac{1}{3}} xQ(3x-5)$

   (c) For which values of $-5 < x < 5$ is the function $Q(x)$ <u>not</u> continuous?

   (d) For which values of $-5 < p < 5$ is $\lim_{x\to p^-} Q(x) \neq Q(p)$?