

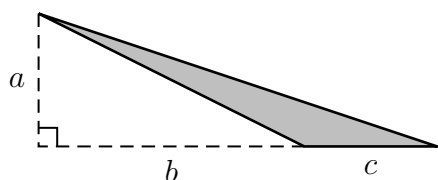
Worksheet Dragon

1. *The Saga of Michael Phelps: Conclusion* Last time we found that Michael Phelps can always make himself dryer by splitting his towel, but there's apparently a limit to how dry he can get. In particular, here are the numbers for not splitting the towel at all and for splitting it into 10,000 pieces:

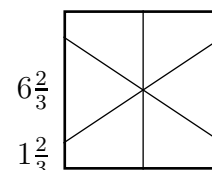
Towel Size	.25	.5	1	2	3	4
wetness (1 piece)	0.8000	0.6667	0.5000	0.3333	0.2500	0.2000
wetness (10,000 pieces)	0.7788	0.6065	0.3679	0.1354	0.0498	0.0183

Cutting into more than 10,000 pieces doesn't seem to make much difference. So for each towel T , there is a wetness $N(T)$ after normal toweling, and there seems to be a "magic number" $M(T)$, which is the *limit* to how dry Michael can get by splitting the towel.

- Make a graph with towel size on the x -axis and wetness on the y -axis. Plot the points you have for $N(T)$, the result of normal toweling, and $M(T)$, the result of split towelling.
- What's the formula for $N(T)$? (We found this previously).
- What kind of function does $M(T)$ look like? Hint: Compare $M(1)$ with $M(2)$.
- Verify your guess by finding a formula that fits the data.
- Using the formula we found last time for splitting the towel into n parts, write a limit equation to express the result in part (d).



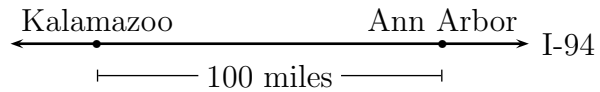
- Find the area of the shaded triangle:
- Cake! We had a breakthrough last time: we found a solution for 6 people. We also had a number of other ideas, some of which involved giving people more than one piece each. (That's legit—as long as everyone gets the same amount of cake and the same amount of frosting.) What other numbers of people can you accommodate?



- dBase™ was a database management system popular on IBM PCs back in the 80s, and still used in some places today. It included a programming language with limited capabilities; for instance, it had regular arithmetic operators (+-*/), but there was no command in the language to take the square root of a number. There were, however, two functions called $\text{LOG}(x)$ and $\text{EXP}(x)$ which produced $\ln(x)$ and e^x , respectively. How could you use them to produce \sqrt{x} ?

5. Kalamazoo is 100 miles west of Ann Arbor along Route 94.

Let $T(x)$ be the temperature in Fahrenheit at a point x miles west of Ann Arbor.



- (a) Define a function A in terms of T so that $A(m)$ is the temperature in Fahrenheit at a point m miles east of Kalamazoo.
- (b) Define a function B in terms of T so that $B(k)$ is the temperature in Fahrenheit at a point k **kilometers** east of Kalamazoo. (1 mile = 1.6 kilometers.)
- (c) Define a function C in terms of T so that $C(k)$ is the temperature in **Celsius** at a point k kilometers east of Kalamazoo.
6. We're still sitting in this cabin, trying to figure out the temperature in Fahrenheit. We have a Celsius thermometer, $33\frac{1}{3}$ RPM record player, a roll of duct tape, a Beatles album from 1967, a stopwatch, and some very cold feet. We know that if c is the temperature we read on the Celsius thermometer, then $f = \frac{9}{5}c + 32$ is the temperature in Fahrenheit. We need to convert from Celsius to Fahrenheit without multiplying or dividing.
7. (This problem appeared on a Winter, 2017 Math 115 exam.) A company designs chambers whose interior temperature can be controlled. Their chambers come in two models: Model A and Model B.

- (a) The temperature in Model A goes from its minimum temperature of -3°C to its maximum temperature of 15°C and returning to its minimum temperature three times each day. The temperature of this chamber at 10 am is 15°C . Let $A(t)$ be the temperature (in $^\circ\text{C}$) inside this chamber t hours after midnight. Find a formula for $A(t)$ assuming it is a sinusoidal function.
- (b) Let $B(t)$ be the temperature (in $^\circ\text{C}$) inside Model B t hours after midnight, where

$$B(t) = 5 - 3 \cos\left(\frac{3}{7}t + 1\right).$$

Find the two smallest positive values of t at which the temperature in the chamber is 6°C . Your answer must be found algebraically. *Show all your work and give your answers in exact form.*

8. (This problem appeared on a Winter, 2013 Math 115 Exam.) The air in a factory is being filtered so that the quantity of a pollutant, P (in mg/liter), is decreasing exponentially. Suppose t is the time in hours since the factory began filtering the air. Also assume 20% of the pollutant is removed in the first five hours.
- (a) What percentage of the pollutant is left after 10 hours?
- (b) How long is it before the pollution is reduced by 50%?