Douglass Houghton Workshop, Section 1, Wed 08/27/25

Worksheet Beatles

1. Last time we found that when Michael Phelps dries himself off with a towel the same size as he is, then he ends up half as wet as he was before. And if the towel is twice as big as he is, he's a third as wet. But then we realized that if he just divides the big towel in half, he can get even drier.

   (a) Assume again that Michael's surface area is $1\,\mathrm{m}^2$, and he starts with 1 liter of water on him. Fill in the table below with his wetness after drying in the normal way, and after drying by splitting the towel into two pieces:

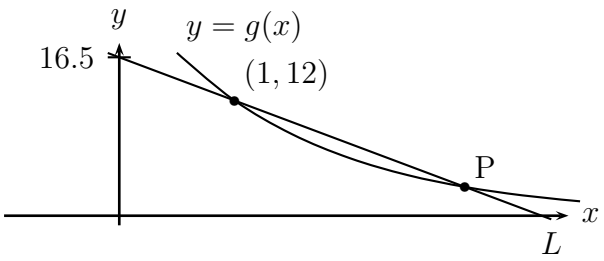
Towel size	Wetness after "normal" towelling	Wetness after "split" toweling
$1\,\mathrm{m}^2$	$1/2\ \ell$	
$2\,\mathrm{m}^2$	$1/3\ \ell$	$1/4\ \ell$
$4\,\mathrm{m}^2$		
$8\,\mathrm{m}^2$		

   (b) What if the towel has surface area T? Find formulas for the two columns in terms of T.

   (c) Can you get him even drier with the same towel?

2. Is it possible to dig a square hole in a triangular plot of land, in such a way that all four corners of the square are on the sides of the triangle?

3. Consider the following graph:



   (a) Find an equation for the line L.

   (b) Find the y-coordinate of the point P, given that its x-coordinate is 3.

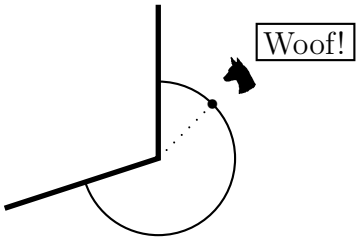
   (c) $g(x)$ is an exponential function. Find a formula for $g(x)$.

4. Last time we found that if c is the temperature we read on the Celsius thermometer, then $f = \frac{9}{5}c + 32$ is the temperature in Fahrenheit.

   Imagine yourself in a cabin in the backwoods of northern Canada. You are very cold from a long day of skiing, and you need to know the temperature in Fahrenheit, so you can tell how much antifreeze to put into your car so that it will start in the morning. Unfortunately, this is Canada, so the only thermometer available is in Celsius, and you are too cold to multiply, so you can't use the formula above. You probably couldn't even add more than two numbers together.

   In the cabin with you are a roll of duct tape, a $33\frac{1}{3}$ RPM record player, a Beatles record from 1967, and a stopwatch, among other things. How can you compute the temperature in Fahrenheit without multiplying or dividing?

5. The security detail at the Pentagon in Arlington, VA, has chained five Rottweilers to each of the five corners of the building. Each chain is 100 feet long. Find the length of the path a single Rottweiler can walk, from one wall to the other, if she is at the end of her chain.



6. Suppose you bake a square cake, 10 inches on a side and 2 inches high. You frost it on the top and all four sides (but not the bottom). How can 6 people divide up the cake so that each gets the same amount of cake and the same amount of frosting? How about 9 people? n people?

7. Suppose that $f(0) = 20$ and $f(3) = 40$.

   (a) Find a formula for $f(x)$, assuming f is linear.

   (b) Find a formula for $f(x)$, assuming f is exponential.

   (c) Find a formula for $f(x)$, assuming f is a quadratic polynomial. How many answers are there? If there are more than one, sketch as many as you can.

8. Write down the algebraic and geometric definitions of even and odd functions.

   (a) What kind of function do you get when you multiply two even functions? Write a proof, using the definitions.

   (b) How about the product of two odd functions?

   (c) Odd times even?

   (d) Odd plus odd, even plus even, odd plus even?

   (e) If a polynmial is odd, what can you say about it?

   (f) What if a polynomial is even?

   (g) A good crossword puzzle has 180-degree symmetry. Prove that if a is the number of across clues and d is the number of down clues, then the numbers a and d are either both even or both odd.