Some relationships from vector analysis

$$\nabla = \left(\hat\imath\,\frac{\partial}{\partial x} + \hat\jmath\,\frac{\partial}{\partial y} + \hat k\,\frac{\partial}{\partial z}\right) : \qquad \text{nabla operator}$$

$$\begin{array}{lllll}
\nabla \cdot \phi & = \text{grad } \phi: & \phi \ \text{ scalar field} & \rightarrow & \text{vector field} \\
 & = \left(\hat\imath\,\frac{\partial}{\partial x}\phi + \hat\jmath\,\frac{\partial}{\partial y}\phi + \hat k\,\frac{\partial}{\partial z}\phi\right) & & & \\
\nabla \cdot \vec{A} & = \text{div } \ \vec{A}: & \vec{A} \ \text{ vector field} & \rightarrow & \text{scalar field} \\
 & = \left(\frac{\partial}{\partial x}A_x \ + \ \frac{\partial}{\partial y}A_y \ + \ \frac{\partial}{\partial z}A_z\right) & & & \\
\nabla \times \vec{A} & = \text{curl } \ \vec{A}: & \vec{A} \ \text{ vector field} & \rightarrow & \text{vector field}
\end{array}$$

$$= \begin{vmatrix} \hat\imath & \hat\jmath & \hat k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \left(\frac{\partial}{\partial y}A_z - \frac{\partial}{\partial z}A_y\right)\hat\imath + \left(\frac{\partial}{\partial z}A_x - \frac{\partial}{\partial x}A_z\right)\hat\jmath + \left(\frac{\partial}{\partial x}A_y - \frac{\partial}{\partial y}A_x\right)\hat k$$

$$\nabla^2 = \nabla \cdot \nabla = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) : \qquad \text{laplacian operator}$$

applied on electric potential U:

$$\nabla^2\, U = \text{div grad } U = 0 \qquad \text{“Laplace equation”}$$

$$\nabla^2\, U = \left(\frac{\partial^2}{\partial x^2}\,U + \frac{\partial^2}{\partial y^2}\,U + \frac{\partial^2}{\partial z^2}\,U\right) = 0$$

<u>Next:</u> $\quad \nabla \times (\nabla \times \vec{A}) = \nabla\,(\nabla \cdot \vec{A}) - \nabla^2\,\vec{A}$

<u>where:</u> $\quad \nabla \cdot \vec{A} = \frac{\partial}{\partial x}A_x + \frac{\partial}{\partial y}A_y + \frac{\partial}{\partial z}A_z \equiv S$

so that $\nabla\,(S) = \hat\imath\,\frac{\partial}{\partial x}S + \hat\jmath\,\frac{\partial}{\partial y}S + \hat k\,\frac{\partial}{\partial z}S$

<u>and</u>

$$\begin{aligned}
\nabla^2\,\vec{A} &= \left(\frac{\partial^2}{\partial x^2}A_x + \frac{\partial^2}{\partial y^2}A_x + \frac{\partial^2}{\partial z^2}A_x\right)\,\hat\imath \\
&+ \left(\frac{\partial^2}{\partial x^2}A_y + \frac{\partial^2}{\partial y^2}A_y + \frac{\partial^2}{\partial z^2}A_y\right)\,\hat\jmath \\
&+ \left(\frac{\partial^2}{\partial x^2}A_z + \frac{\partial^2}{\partial y^2}A_z + \frac{\partial^2}{\partial z^2}A_z\right)\,\hat k
\end{aligned}$$