Physics 340 **Mathematics Review - Solutions** **6-September-00**

Name (please print):____

Fill in the spaces below to the best of your ability. You may consult textbooks or other sources as needed. Write your answer in the simplest form.

1. Laws of exponents:

$$(x^a) \times (x^b) = x^{(a+b)} \qquad (x^a)^b = x^{(ab)}$$

2. Basic Trigonomic functions and identities:

$$\sin^2\theta + \cos^2\theta = 1 \qquad \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

3. Derivatives and integrals of powers:

$$\frac{d}{dx}(x^n) = nx^{(n-1)}, n \in R \qquad \int x^n\,dx = \frac{x^{(n+1)}}{n+1}, n \neq -1$$

4. Derivatives and integrals of logarithms and exponentials:

$$\frac{d}{dx}(e^{\alpha x}) = \alpha e^{\alpha x} \qquad \int e^{\alpha x}\,dx = \frac{1}{\alpha}e^{\alpha x}$$

$$\frac{d}{dx}\ln(|x|) = \frac{1}{x} \qquad \int \ln(|x|)\,dx = x(\ln|x| - 1)$$

5. Derivatives and integrals of trigonomic functions:

$$\frac{d}{d\theta}\sin\theta = \cos\theta \qquad \int \sin\theta\,d\theta = -\cos\theta$$

$$\frac{d}{d\theta}\cos\theta = -\sin\theta \qquad \int \cos\theta\,d\theta = \sin\theta$$

6. The Product Rule (Leibniz's rule):

$$\frac{d}{dx}[f(x)g(x)] = g\frac{df}{dx} + f\frac{dg}{dx}$$

7. Chain Rule (derivative of composite functions):

$$\frac{d}{dx}[f[g(x)]] = \frac{df}{dg}\frac{dg}{dx} \qquad\qquad \frac{d}{d\theta}\cos(2\theta) = -2\sin(2\theta)$$

8. Fundamental Theorem of the Calculus: if $f(x)$ and $F(x)$ are functions such that $\frac{dF}{dx} = f(x)$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

9. Implicit Differentiation: If $x^2 + y^2 = r^2$, with r a constant, obtain

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{r^2 - x^2}\right) = -\frac{x}{y}$$

10. Taylor's Theorem (series expansion):

$$f(x_0 + \epsilon) = \sum_{n=0}^{\infty}\left[\quad \frac{1}{n!}\frac{d^n f}{dx^n}\bigg|_{x=x_0} \quad\right]\epsilon^n$$

11. Partial Derivatives: If $\phi(x, y, z) = x^3y^2z$ then

$$\frac{\partial\phi}{\partial x} = 3x^2y^2z$$

$$\frac{\partial\phi}{\partial y} = 2x^3yz$$

$$\frac{\partial\phi}{\partial z} = x^3y^2$$

12. Gradient: If $\phi(x, y, z) = x^3y^2z$ (as above) then

$$\nabla\phi = (3x^2y^2z)\hat{\imath} + (2x^3yz)\hat{\jmath} + (x^3y^2)\hat{k}$$

13. Divergence: Given $\vec{A} = 2x\hat{\imath} + 3y\hat{\jmath} + 5z\hat{k}$,

$$\nabla\cdot\vec{A} = 2 + 3 + 5 = 10$$

14. Curl: Given $\vec{B} = 7z\hat{\imath} + 2x\hat{\jmath} + 11y\hat{k}$,

$$\nabla\times\vec{B} = 11\hat{\imath} + 7\hat{\jmath} + 2\hat{k}$$