

Statement of Interest

Dear REU coordinators,

My name is Lena Ji. I am currently a third-year studying mathematics at Columbia University, and I would love to have the opportunity to participate in SMALL this summer. I am particularly interested in the commutative algebra project because I have really loved the algebra I have seen thus far, but most importantly I am excited to see as many new concepts and ideas as possible.

I became especially interested in algebra while taking the second half of Columbia's abstract algebra sequence last semester, which covered rings, ideals, fields, and Galois theory. It was amazing to see how much machinery was constructed to solve what might at first glance appear to be simple polynomial equations, and to then see how these techniques generalized to explain fundamental connections between concepts as seemingly different as number fields and permutations. After the course ended, I continued working on Dummit and Foote's *Abstract Algebra*, and read through several topics not covered in the formal course, including tensors and modules.

This semester, I am doing a supervised reading course on Atiyah and MacDonald's *Introduction to Commutative Algebra* with Professor Aise Johan de Jong. So far, we have gone over rings and ideals with a much more intense treatment than the first time I saw the material, which has been quite exciting; it's helping me solidify my understanding while relating and weaving in algebraic geometry. Another course that I am particularly enjoying is Algebraic Number Theory, which is following J. S. Milne's notes and also structured similarly to a reading course. It is currently covering Dedekind domains and factorization, and it's very interesting to see how commutative algebra is used in a number theoretic context. The semester has only just started, but I'm looking forward to learning much more in the upcoming months.

My current perspective on mathematics and abstraction is something I would have never anticipated several years ago. When I first entered college, I initially planned to major in economics; however, after taking Linear Algebra last year, I was exposed for the first time to math that was not purely mechanical, and enjoyed it so much that I decided to switch directions to mathematics. Up until that point, the limited amount of math I had seen followed a formula or specific outlined procedure given to find a solution. What I found new and intriguing was not so much the final arrival at a nice, neat answer, but more of the process of how to get there. Even when using different approaches to solve a problem, the different methods would have some connection and the end results would be compatible with each other – I wanted to find out precisely what that connection was and why it existed. More important than the fact that different fields tie together in a coherent, unified manner is discovering how they work together and why that is the case – and finding out what else that leads to. Seeing the same themes and organizational structures reappearing in unexpected contexts, for example the manner in which field extensions and subgroups of a Galois group parallel subgroups of the fundamental group and covering spaces, is just incredible. Although my interest in math may have begun later than others', I am very keen to learning more and excited to work hard to do so.

I hope to become completely immersed in math and also gain valuable research experience this summer through participating in SMALL. This past summer, I worked on an REU project at Columbia on geometric group theory. Specifically, we studied algebraic invariants, including thickness, of right-angled Coxeter groups associated to finite simplicial graphs. Through this program, I was able to get a first taste of research, as well as read a lot of algebra and get to know others who were passionate about math. This coming summer, I would like to go to Williams instead to gain exposure to a different setting and meet new people. The prospect of being able to work together in such a close proximity with other highly motivated students is very appealing. The fact that that the different research groups live in the same house is also exciting, since it fosters a strong sense of community and will allow more interaction and exchange of ideas with students doing research in other areas of math. In the future, I hope to

continue learning and experiencing as much math as possible. I would like to do so by pursuing graduate studies, and I believe that SMALL would help me to do so. I would like to end this statement with a quote from Paul Halmos's *I Want to Be a Mathematician*: "I am not recommending or insisting that you love mathematics. I am not issuing an order: 'If you want to be a mathematician, start loving mathematics forthwith' —that would be absurd. What I am saying is that the love of mathematics is a hypothesis without which the conclusion doesn't follow." I am slowly but surely working along the path to arriving at that conclusion. Thank you so much for your time and consideration!

Best regards,
Lena