My interest in the DIMACS REU stems from my strong affinity for physically and biologically motivated problems in mathematics. Several of my research and classroom experiences revolve around this theme. Most recently, I worked on a mathematical model for special cells in the bone known as osteocytes with Madison Albert and Drs. Jared Barber and Luoding Zhu as part of the Summer 2021 Indiana University-Purdue University Indianapolis (IUPUI) applied mathematics REU. The overarching goal of the project was to create a three-dimensional model of an osteocyte and its surrounding fluid to better understand how the cells sense and respond to forces. The solid components of the cell were modeled as a network of damped springs, and I worked on computing the resulting forces on the cell. After computing the forces, we obtained a large system of differential equations that we numerically integrated using Matlab to determine the cellular dynamics in response to compression forces. We also used Matlab to visualize these dynamics. The results of the REU are currently being compiled into a paper that we plan to publish this year and I have also presented the results at 4 different conferences.

In Spring 2021, I worked on the more macroscopic problem of modeling the motion of aquatic animals such as archer fish at a water-air interface as part of Dr. Leah Mendelson's Flow Imaging Lab at Mudd (FILM). I focused on modeling the oscillatory motion of a tailfin as an inverted pendulum at the interface between two media. In order to tackle the original problem, which was unwieldy and involved several degrees of freedom, I analyzed the dynamics of a system with reduced degrees of freedom. I used Lagrangian mechanics to tackle this complex problem. Since a pendulum at an interface feels different forces from each of the media, I separated the pendulum into a water-submerged section and an air-submerged section. This distinction made the problem more tractable, and I was able to derive the equations of motion of the system and conduct an equilibrium point and stability analysis.

In Fall 2020, I explored problems in the calculus of variations through my Theoretical Mechanics class, as well as my independent study of graduate level calculus of variations under the mentorship of Dr. Dagan Karp. My independent research complemented the class by focusing heavily on the mathematical rigor of many of the results used in the Theoretical Mechanics class. Using Mark Kot's "A First Course in the Calculus of Variations" as a guide, we studied topics such as geodesics, Lagrangian and Hamiltonian mechanics, Lagrangian densities, and isoperimetric and holonomic constraints. These tools were used in a computational project on the Brachistochrone problem with kinetic friction. The conventional Brachistochrone problem searches for the trajectory that minimizes the time of travel between two points on a vertical plane and assumes that there are no non-conservative forces like friction and drag. I decided to examine a modification of the problem that accounts for kinetic friction. The first half of the project used calculus of variations to solve for the shape of the trajectory, while the second half of the project used Lagrangian analysis, with the help of Mathematica, to analyze the dynamics of the system over time.

The DIMACS REU is a perfect opportunity to apply the mathematical and computational skills I have gained from previous experiences to new and interesting problems in applied mathematics. I particularly resonate with Dr. Shadi Tahvildar-Zadeh's project on one-dimensional mathematical physics. The idea of tackling a huge problem, finding a "theory of everything", by studying a smaller but equally useful and interesting one-dimensional problem reminds me of the approach I used with my research at FILM. The project would also be an exciting opportunity to expand my knowledge of mechanics into the quantum mechanical realm, a field I haven't explored in a research setting before. On the biological side, I am looking forward to exploring fields outside the realm of biomechanics through the numerous biomathematics projects at DIMACS. Furthermore, the opportunity to work with faculty at large research institutions with doctorate programs would be immensely valuable as I prepare for PhD programs in Applied Mathematics.