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Research Statement

My first taste of mathematical research came when I spent summer 2008 at the Research Science Institute at MIT working on a problem in algebraic graph theory, and presented my work at several high school science fairs. However, my most significant research has taken place at Duke. Since spring semester of 2012 I have been working in algebraic knot theory with Professor Christopher Cornwell. I have been studying an constructino called knot contact homology, which is a tensor-algebra-valued invariant of knots. From this tensor algebra it is common to define a two-variable polynomial called the augmentation polynomial;as the name suggests, this polynomial measures when there exists algebra maps from knot contact homology into the complex numbers. I obtained a complete characterization of the augmentation polynomials of $(2, k)$-torus knots, which was the first calculation of the augmentation polynomial of an infinite family of knots. This calculation was achieved via elimination theory; I analyzed the form of the determinant of the Sylvester matrix of two relations obtained from knot contact homology, and obtained a bound on the number of augmentations of certain forms in the two variables; previous knowledge of augmentations of $(2, k)$-torus knots established the form.

Currently I'm trying to understand how augmentations of a knot's contact homology arise as images of representations $\rho : \pi_1(\mathbb{R}^3 - K) \to GL_k\mathbb{C}$ where the peripheral system $\langle l, m\rangle$ of the knot are sent to diagonal matrices of a certain form. In particular the existence or non-existence of such representations in certain dimensions is closely tied to certain classical knot invariants such as meridianal rank or bridge number; I'm trying to improve the known bounds and establish the reverse directions of known relations.

In terms of larger-scale interests, I'm most fascinated by arithmetic topology. This project attempts to study the analogies one gets by trying to look at $Spec(\mathbb{Z})$ as a 3-manifold, as is suggested by Artin-Verdier duality. In particular there is a large dictionary (called the Mazur-Kapranov-Reznikov, or MKR, dictionary) that provides an analogy between knots and links in 3-manifolds and ideals in the rings of algebraic integers of a number field; for example, under this dictionary a peripheral system of a knot corresponds to a Frobenius automorphism and a monodromy generating the tame inertial group. This is personally interesting because, while I've gotten into knot theory mostly in college, number theory was easily my favorite and best subject in high school competitions, and the idea of a high-level link between the two is motivating as a higher-level, and perhaps more fruitful, way to view knot theory.