MEASUREMENT OF THE $WW\gamma$ and WWZ COUPLINGS IN THE PROCESS $e^+e^- \to \ell\nu q\overline{q}'$

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ABSTRACT

We studied the sensitivity of the process $e^+e^- \to \ell\nu q\overline{q}'$ to anomalous trilinear gauge boson couplings of the $WW\gamma$ and WWZ vertices at the center of mass energies $\sqrt{s} = 500$ GeV and 1 TeV. The bounds for the couplings we obtained result from an analysis of a five dimensional angular differential cross section. In our calculations we included all tree level Feynman diagrams contributing to the final state as well as the finite widths of the vector bosons. Both unpolarized and polarized beams were considered. We found that the 500 GeV measurements will be at the level of loop contributions to the couplings and may show hints of new physics while the 1 TeV should be sensitive to new physics at the loop level. We also explored $\ell\nu q\overline{q}$ final states off the W resonance and found that useful information could be extracted from this region of phase space.

1 Introduction

One of the important goals of the Next Linear Collider (NLC) is to make precision measurements of W boson properties including W-boson interactions with the photon and Z^0. The measurements of the trilinear gauge boson vertices (TGV's) provide a stringent test of the gauge structure of the standard model[1, 2]. The current measurement of these couplings performed by the CDF and D0 collaborations [3] are rather weak. Although the measurements at the Large Hadron Collider should improve these limits considerably it is expected that measurements at high energy e^+e^- colliders will surpass those at the hadron colliders.

Probably the most useful of the $e^+e^- \to W^+W^-$ channels for these measurements is $e^+e^- \to \ell\nu q\overline{q}'$. With only one unobserved neutrino this channel has several advantages: it can be fully reconstructed using the constraint of the initial beam energies, the W^+ and W^- can be discriminated using lepton charge identification, it does not have the QCD backgrounds that plague the fully hadronic decay modes, and it offers much higher statistics than the fully leptonic modes.

In this paper we examine the sensitivity of this channel to anomalous $WW\gamma$ and WWZ^0 gauge boson couplings. We study this process for the centre of mass energies $\sqrt{s} = 500$ and 1000 GeV appropriate to the NLC. In our calculations we included all tree level diagrams to the four fermion final states using helicity amplitude techniques and we considered the finite widths of the vector bosons. To determine the sensitivity of the channel to anomalous gauge boson couplings we examined numerous distributions [4]. We also looked at the question of how important the non-resonant diagrams are in these calculations and how much information about the TGV's can be found in the off-resonance production. Using helicity amplitudes we are able to study the usefulness of initial state polarization in extracting the TGV's.

In the next section we discuss the effective Lagrangian parametrization and make introductory remarks on our calculations. In section 3 we present and discuss our results. We summarize our conclusions in section 4.

2 Calculations

2.1 Parametrization

In order to describe TGV's we use the most general parametrization possible that respects Lorentz invariance, electromagnetic gauge invariance and CP invariance [5, 6, 7] . These constraints leave us with five free independent parameters for the $WW\gamma$ and WWZ vertices and the effective interaction lagrangian is given by

$$\mathcal{L}_{WWV} = -ig_V \left\{ g_1^V (W^+_{\mu\nu} W^{-\mu} - W^{+\mu} W_{\mu\nu}) V^\nu + \kappa_V W^+_\mu W^-_\nu V^{\mu\nu} + \frac{\lambda_V}{M_W^2} W^+_{\lambda\mu} W^{-\mu}_\nu V^{\nu\lambda} \right\} \quad (1)$$

where the subscript V denotes either a photon or a Z^0, V^μ and W^μ represents the photon or Z^0 and W^- fields respectively, $W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$ and $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ and M_W is the W boson mass. (g_1^γ is constrained by electromagnetic gauge invariance to be equal to 1.) At tree level the standard model requires $g_1^Z = \kappa_V = 1$ and $\lambda_V = 0$.

This approach has become the standard parametrization used in phenomenology making the comparison of the sensitivity of different measurements to the TGV's straightforward. Later in the paper we show bounds that could be imposed on these five parameters measuring the $e^+e^- \to \ell\nu q\overline{q}'$ process at NLC machines. We also calculate the sensitivity to the L_{9L} and L_{9R} parameters [4] that appear in the chiral Lagrangian parametrization of the effective TGV's.

2.2 Initial remarks and conditions

There are 10 (20) diagrams contributing to the $\mu^\pm\nu_\mu q\overline{q}'$ ($e^\pm\nu_e q\overline{q}'$) final state [4]. Of these there are two (three) diagrams which contain TGV's in the muon (electron) mode of the reaction. To include finite width effects we used vector boson propagators of the form $(s - M_V^2 + i\Gamma_V M_V)^{-1}$ which yields a gauge invariant result. We used the CALKUL helicity amplitude technique [8] to obtain expressions for the matrix elements and performed the phase space integration using Monte Carlo techniques. To obtain numerical results we used the values $\alpha = 1/128$, $\sin^2\theta = 0.23$, $M_Z = 91.187$ GeV, $\Gamma_Z = 2.49$ GeV, $M_W = 80.22$ GeV, and $\Gamma_W = 2.08$ GeV. In our results we included two generations of quarks and took them to be massless. In order to take into account finite detector acceptance we require that the lepton and quarks are at least 10 degrees away from the beam and have at least 10 GeV energy (with exception of Table 1).

3 Results

3.1 Total cross section

We calculate total cross sections as a function of $\sqrt{s}$ for both lepton modes [4]. The values for energies of 175 GeV, 500 GeV, and 1 TeV are given in Table 1. In general the electron mode has a larger cross-section than the muon mode. The difference is small at 175 GeV but becomes increasingly larger at higher energy as the t-channel photon exchange becomes increasingly important, reaching a factor of 5 at 1 TeV.

Imposing cuts on the invariant masses of the $\ell\nu$ and $q\overline{q}'$ pairs we can assess contributions of the non-resonant diagrams to the total cross section. For the muon mode the invariant mass cuts reduce the cross-section by 10% to 20% depending on $\sqrt{s}$ and irrespective of whether the cut is on $M_{\ell\nu}$ or $M_{q\overline{q}}$. The relatively small effect of these cuts verifies the dominance of the resonant diagrams on the total cross section. For the electron mode the cross section with the cut on $M_{q\overline{q}'}$ is significantly larger than the cross section with the cut on

Table 1: *Cross-sections for $e^+e^- \to \mu^+\nu_\mu q\overline{q}'$ and $e^+e^- \to e^+\nu_e q\overline{q}'$ including cuts on the invariant masses of the outgoing fermion pairs, $M_{\ell\nu}$ and $M_{q\overline{q}'}$. A 10^o cut away from the beam is imposed on charged final state fermions and no cut on their energy. The cross-sections are given in pb.*

$\sqrt{s}$ (GeV)	ℓ	no cut	$\lvert M_{q\overline{q}} - M_W\rvert < 5$ GeV	$\lvert M_{\ell\nu} - M_W\rvert < 5$ GeV	both cuts
175	μ	1.10	1.00	1.00	0.91
	e	1.15	1.04	1.01	0.91
500	μ	0.39	0.34	0.34	0.29
	e	0.62	0.53	0.34	0.29
1000	μ	0.077	0.063	0.064	0.052
	e	0.44	0.39	0.064	0.052

$M_{e\nu}$. This can be attributed to poles in the single W production t-channel photon exchange diagrams. With appropriate kinematic cuts this can be used to study single-W production.

Despite the relative smallness of the off-resonance contributions to the muon mode they still contribute up to 30% of the cross section at 1 TeV. Clearly, they must be properly included when making high precision tests of standard model processes. For the electron mode they are even more important and are interesting in the context of single W production.

3.2 Distributions

We examined numerous distributions and applied various kinematic cuts to identify the observables and the regions of phase space that are the most sensitive to anomalous couplings. Details can be found in ref. [4]. Here we summarize the most interesting points and our conclusions.

Any disruption of the delicate gauge theory cancellations leads to large changes to the Standard model results. For W_L production amplitudes the enhancements can be a factor of (s/M_W^2). Because it is the longitudinal W production which is the most sensitive to anomalous couplings, and because the cross section is dominated by transverse W production it is crucial to disentangle the W_L from the W_T *background*. The angular distributions of the W decay products provide the most convenient tool for doing so.

To investigate the sensitivity of the $e^+e^- \to \ell^\pm\nu_\ell q\overline{q}'$ process to the anomalous couplings we used a combined distribution of five angular observables [4, 9, 10] : Θ, θ_{qq}, ϕ_{qq}, $\theta_{\ell\nu}$, and $\phi_{\ell\nu}$, where Θ is the W^- scattering angle with respect to the initial e^+ direction, θ_{qq} is the polar decay angle of the q in the W^- rest frame using the W^- direction as the quantization axis, ϕ_{qq} is the azimuthal decay angle of the q in the W^- rest frame, and $\theta_{\ell\nu}$ and $\phi_{\ell\nu}$ are the analogous angles for the lepton in the W^+ rest frame. There is an ambiguity in measurement

Table 2: *Sensitivities to anomalous couplings for the various parameters varying one parameter at a time. The combined values are obtained by combining the four lepton modes (e^-, e^+, μ^-, and μ^+) and two generations of light quarks (ud, cs). The results are 95% confidence level limits.*

mode	δg_1^Z	$\delta\kappa_Z$	$\delta\kappa_\gamma$	λ_Z	λ_γ	L_{9L}	L_{9R}
$\sqrt{s} = 500$ GeV, L=50 fb^{-1}, $\lvert M_{\ell\nu(q\overline{q})} - M_W\rvert < 10$ GeV							
μ	±0.020	±0.007	±0.005	±0.005	±0.006	$^{+2.2}_{-2.1}$	$^{+4.6}_{-4.2}$
e	$^{+0.019}_{-0.020}$	±0.007	±0.005	±0.005	±0.006	$^{+2.2}_{-2.1}$	$^{+4.6}_{-4.2}$
combined	±0.0095	±0.0035	±0.0025	±0.0025	±0.0025	$^{+1.1}_{-1.0}$	$^{+2.2}_{-2.1}$
$\sqrt{s} = 1$ TeV, L=200 fb^{-1}, $\lvert M_{\ell\nu(q\overline{q})} - M_W\rvert < 10$ GeV							
μ	±0.01	±0.002	±0.001	±0.002	±0.002	$^{+0.61}_{-0.62}$	$^{+1.3}_{-1.1}$
e	±0.01	±0.002	±0.001	±0.002	±0.002	$^{+0.61}_{-0.62}$	$^{+1.3}_{-1.1}$
combined	±0.0054	±0.001	±0.0006	±0.0008	±0.0008	±0.28	$^{+0.62}_{-0.56}$

of the $q\overline{q}$ pair observables since we cannot tell which hadronic jet corresponds to the quark and which to the antiquark. We therefore include both possibilities in our analysis. Each of the variables was divided into four bins so that the entire phase space was divided into $4^5 = 1024$ bins. To extract the information about the TGV's we used the maximum likelihood method using Poisson statistics because with this many bins some will not be very populated with events. The results we obtained are based solely on the statistical errors using the integrated lumininosity we assumed for the various cases.

A thorough analysis of gauge boson couplings would allow all five parameters in the Lagrangian to vary simultaneously to take into account cancellations (and correlations) among the various contributions. This approach is impractical, however, due to the large amount of computer time that would be required to search the parameter space. Instead we found 2-dimensional *exclusion* contours for a selection of parameter pairs to give a sense of the correlations. Because of limited space, here we show only limits obtained for individual parameters when all the other anomalous parameters are kept fixed at their Standard model values (Table 2). In Figure 1 we show a few representative contours for $\sqrt{s} = 500$ GeV. More contours can be found in [4].

It was pointed out that the angular distributions with different initial electron polarizations have different dependences on anomalous couplings [4, 11]. We calculated contours for $e^-_L e^+$ and $e^-_R e^+$ initial states [4]. An important difference between the two polarizations is that the cross-section with left handed electrons is about an order of magnitude larger than for right handed electrons. At the same time the right-handed cross-section is significantly more sensitive to anomalous couplings than the left-handed cross-section. There is therefore a tradeoff between sensitivity and statistics so that in most cases the single parameter bounds obtainable for the two polarizations are comparable. In general, the unpolarized contours are

Table 3: *Sensitivities to anomalous couplings based on off-resonance cross sections varying one parameter at a time. The values are obtained by combining the four lepton modes (e^-, e^+, μ^-, and μ^+) and two generations of light quarks (ud, cs). The results are 95% confidence level limits. A dash signifies that the bound is signficantly weaker than the others.*

Initial State	cut	δg_{1Z}	$\delta\kappa_Z$	$\delta\kappa_\gamma$	λ_Z	λ_γ	L_{9L}	L_{9R}
$\sqrt{s} = 500$ GeV, L=50 fb^{-1}								
e^-_L	$M_{\ell\nu} > M_W + 15$ GeV	$^{+0.13}_{-0.18}$	$^{+0.056}_{-0.074}$	$^{+0.012}_{-0.012}$	$^{+0.026}_{-0.034}$	$^{+0.026}_{-0.041}$	$^{+6.9}_{-7.2}$	$^{+8.9}_{-9.2}$
e^-_R	$M_{\ell\nu} > M_W + 15$ GeV	$^{+0.08}_{-0.24}$	$^{+0.03}_{-0.10}$	$^{---}_{-0.018}$	$^{+0.023}_{-0.033}$	$^{+0.034}_{-0.023}$	$^{---}_{---}$	$^{---}_{---}$
e^-	$M_{\ell\nu} > M_W + 15$ GeV	$^{+0.15}_{-0.21}$	$^{+0.07}_{-0.10}$	$^{+0.016}_{-0.017}$	$^{+0.031}_{-0.038}$	$^{+0.033}_{-0.043}$	$^{+10}_{-10}$	$^{+13}_{-13}$
$\sqrt{s} = 1$ TeV, L=200 fb^{-1}								
e^-_L	$M_{\ell\nu} > M_W + 15$ GeV	$^{+0.038}_{-0.085}$	$^{+0.013}_{-0.014}$	$^{+0.005}_{-0.005}$	$^{+0.002}_{-0.002}$	$^{+0.003}_{-0.003}$	$^{+2.4}_{-2.5}$	$^{+3.8}_{-3.9}$
e^-_R	$M_{\ell\nu} > M_W + 15$ GeV	$^{+0.062}_{-0.102}$	$^{+0.010}_{---}$	$^{---}_{-0.010}$	$^{+0.007}_{-0.007}$	$^{+0.007}_{-0.007}$	$^{---}_{---}$	$^{---}_{-5.9}$
e^-	$M_{\ell\nu} > M_W + 15$ GeV	$^{+0.047}_{-0.096}$	$^{+0.018}_{-0.021}$	$^{+0.007}_{-0.007}$	$^{+0.003}_{-0.003}$	$^{+0.003}_{-0.003}$	$^{+3.4}_{-3.5}$	$^{+5.4}_{-5.6}$

aligned along the e^-_L contours, which is not too suprising considering that $\sigma(e^-_L)$ dominates over the right-handed contribution in the unpolarized cross-section. For some combinations of the parameters the different polarizations give much different correlations which could be useful for disentangling the nature of anomalous TGV's if deviations were observed.

3.3 Off-resonance production

In this section we explore the information potential available from $\ell\nu q\overline{q}$ final states off the W resonance. We calculated invariant mass distributions ($M_{\mu\nu}$, $M_{e\nu}$, and $M_{q\overline{q}'}$) of final state fermions with various polarizations of the initial electron [4]. Varying TGV's parameters and looking at the response of these distributions we found that there is considerable sensitivity to anomalous gauge boson couplings even when the fermion pairs do not originate from real W production.

To quantitatively demonstrate that there is considerable information in the non-resonant production we find limits for the TGV's parameters based on the total cross section while imposing the cuts $M_{ff'} < M_W - 15$ GeV and $M_{ff'} > M_W + 15$ GeV. Here, the $M_{ff'}$ is the invariant mass of the final state fermion pairs and ff' stands for either $\ell\nu$ or $q\overline{q}$. These give rise to a large number of possibilities so we only present the "best" case when the four possible final states are combined for each energy. The most significant bounds are presented in Table 3. We warn that our analysis is not a rigorous one. In particular we do not consider possible backgrounds to non-resonant events and do not make any effort to optimize our cuts to enhance deviations from SM results.

From the above results it is clear that, although the constraints that could be obtained from off-resonance production are not as tight as those obtained from on-shell W production,

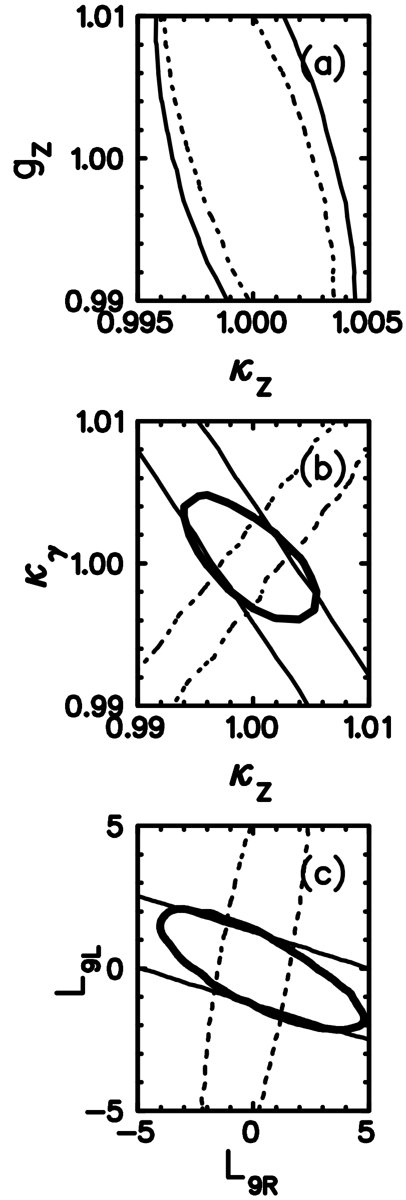

Figure 1: 95% C.L. exclusion contours for sensitivity to anomalous couplings for unpolarized (the heavy solid line) and polarized initial state electrons with $\sqrt{s} = 500$ GeV, combining all four lepton charge states. The solid curves are for $e_L^- e^+$, the dashed curves for $e_R^- e^+$. The integrated luminosity is $L = 25$ fb^{-1} per polarization, for unpolarized electrons $L = 50$ fb^{-1}.

there is nevertheless considerable information contained in these events. It appears to us that the method that makes optimal use of each event is to calculate the probability of each event, irrespective of where it appears in phase space, and compute a likelihood function for the combined probabilities. The only cuts that should be included are those that represent detector acceptance and that are introduced to eliminate backgrounds.

4 Conclusions

We performed a detailed analysis of the measurement of tri-linear gauge boson couplings in the process $e^+e^- \to \ell^\pm \nu q\overline{q}$. To gauge the sensitivity of this process to anomalous gauge boson couplings we used the W decay distributions as a polarimeter to distinguish the longitudinal W modes, which are more sensitive to anomalous coupings, from the transverse modes. We implemented this through the use of a quintic differential cross section, with each angular variable divided into 4 bins, and then calculating the likelihood function of non-standard model couplings as compared to the standard model. The 500 GeV NLC measurements are sensitive enough that they should be sensitive to loop contributions to the TGV's while the 1 TeV will be able to measure such effects.

We studied the sensitivity of the off-mass shell cross sections to anomalous couplings by imposing kinematic cuts on the invariant mass distributions of the outgoing fermion pairs. A cursory analysis found that the off-resonance cross section is relatively sensitive to anomalous couplings and that useful information could be extracted from this region of phase space.

The optimal strategy to maximize the information contained in each event is to construct a likelihood function based on the four vector of each of the outgoing fermions on an event by event basis, putting them through a realistic detector simulation. This would make the best use of the information whether it be on the W resonance or not. Kinematic cuts should only be introduced to reduce backgrounds. Since the precision of these measurements is beyond the level of loop induced radiative corrections it is crucial that radiative corrections are well understood and included in event generators used in the study of these processes.

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