

Public Policy 741 Handout
Derivation of the Fisher Effect

NOTE: This handout is intended to help students who are interested in understanding the derivation of the Fisher Effect presented in class. If you find this material difficult, do not despair, it is not necessary to understand the mathematics behind the model in order to be able to *use* the model to understand interest rate behavior.

We can do the derivation either in percentage changes or in natural logs – I did it in terms of percentages in class – here I do it with natural logs. For a discussion of how the two are related see the last page of this handout.

Recall, some of the basic rules of natural logs (ln):

$$\ln(xy) = \ln(x) + \ln(y); \quad \ln\left(\frac{x}{y}\right) = \ln(x) - \ln(y)$$

Notation:

- a k-period-ahead variable is given a (time) subscript, t+k.
- lower case letters denote the natural logarithm (e.g. $\ln(P)=p$), with the exception of the k-period-ahead interest rate (i_{t+k}).
- Foreign variables are denoted with an asterisk *.

Absolute Purchasing Power Parity (PPP) specifies the domestic currency per foreign currency spot exchange rate (S) to be equal to the domestic price index (P) divided by the foreign price index (P*):

$$S = P/P^*.$$

The log-linearized version of PPP (including time subscripts) is:

$$s_t = p_t - p_t^*$$

Relative Purchasing Power Parity states that the log (or percentage) change in the exchange rate between two currencies equals the difference between the log (or percentage) change in national price levels (over a specific time period). In log-linearized form relative PPP is:

$$[s_{t+k}^e - s_t] = [p_{t+k}^e - p_t] - [p_{t+k}^{*e} - p_t^*]$$

Now, substitute our notation for inflation, π^e and π^{*e} for the price change terms:

$$[s_{t+k}^e - s_t] = \pi_{t+k}^e - \pi_{t+k}^{*e}$$

Uncovered Interest Parity (UIP) states the expected future spot rate is the current spot rate multiplied by the (euro-currency) interest differential:

$$S_{t+k}^e = S_t \frac{(1 + i_{t+k})}{(1 + i_{t+k}^*)}$$

In log-linearized form UIP can be approximated with (see the handout on the forward market for more details on this):

$$S_{t+k}^e - S_t = i_{t+k} - i_{t+k}^*$$

If we substitute UIP into the relative PPP equation – we get the *Fisher Effect* which tells us that, all else equal, a rise in a country's expected inflation rate will eventually cause an equal rise in the interest rate that deposits of its currency offers:

$$\pi_{t+k}^e - \pi_{t+k}^{*e} = i_{t+k} - i_{t+k}^*$$

The Fisher Effect gives us the (long-run) prediction that a domestic currency depreciates when its interest rate rises relative to foreign interest rates (if the underlying cause of the interest rate rise is inflation).

The Relationship Between Percentage Change and Log Differences

Under what conditions are log differences valid approximations to percentage changes? Or, specifically, when will the following expression hold:

$$\frac{S_{t+k}^e - S_t}{S_t} \approx \ln(S_{t+k}^e) - \ln(S_t)$$

where $\approx$ denotes approximately equal.

Begin by considering the function $f(x) = \ln(1+x)$. Evaluate a first-order Taylor approximation to this function in the region in which x is close to zero. (A first order Taylor approximation to $f(x)$ in the neighborhood of $x \approx 0$ is: $f(0) + f'(x)x$.) The first derivative of the function is: $f'(x) = 1/(1+x)$, so the first-order Taylor approximation is:

$$f(x) = \ln(1+x) \approx \ln(1) + \frac{x}{1+x}$$

Since $\ln(1)=0$, the value of the function when x is close to zero is:

$$\lim_{x \rightarrow 0} \ln(1+x) = x.$$

Now consider the ratio S_{t+k}^e / S_t . This ratio equals:

$$1 + \left(\frac{S_{t+k}^e - S_t}{S_t} \right)$$

As long as the expression in parentheses is small, reflecting a small expected change in the spot exchange rate, we can replace x by this expression in our formula above, so that:

$$\ln \left(\frac{S_{t+k}^e}{S_t} \right) \approx \frac{S_{t+k}^e - S_t}{S_t}$$

And, since $\ln(S_{t+k}^e / S_t) = \ln(S_{t+k}^e) - \ln(S_t)$, the difference of logarithms is a valid approximation to small percentage changes.