Proceedings of the ASME 2020 Dynamic Systems and Control Conference
DSCC
October 4 – 7, 2020, 2020, Pittsburgh, USA

DSCC2020-23921

AN INTEGRATIVE BEHAVIORAL-BASED PHYSICS-INSPIRED APPROACH TO TRAFFIC CONGESTION CONTROL

Hossein Rastgoftar
Department of Mechanical Engineering
Villanova University
Villanova, PA 19085
Email: hossein.rastgoftar@villanova.edu

Jean-Baptiste Jeannin
Department of Aerospace Engineering
University of Michigan
Ann Arbor, MI, 48109 USA
Email: jeannin@umich.edu

Ella Atkins
Department of Aerospace Engineering
University of Michigan
Ann Arbor, MI, 48109 USA
Email: ematkins@umich.edu

ABSTRACT

This paper offers an integrative behavioral-based physics-inspired approach to model and control traffic congestion in an efficient manner. While existing physics-based approaches commonly assign density and traffic flow states with the Fundamental Diagram, this paper specifies the flow-density relation using past traffic behavior (intent) recorded over a time sliding window with constant horizon length. With this approach, traffic coordination trends can be consistently learned and incorporated into traffic planning. This is integrated with mass conservation law (continuity) to model traffic coordination as a probabilistic process and obtain traffic feasibility conditions using linear temporal logic. By spatial discretization of a network of interconnected roads (NOIR), the NOIR is represented by a graph with inlet boundary nodes, outlet boundary nodes, and interior nodes. The paper offers a boundary control approach to manage congestion through the inlet boundary nodes. More specifically, model predictive control (MPC) is applied to control traffic congestion through the boundary of the traffic network. Therefore, the optimal boundary inflow is assigned as the solution of a constrained quadratic programming problem with equality and inequality constrained. The simulation results shows that the proposed MPC boundary controller can successfully control the traffic through the inlet boundary nodes where traffic reaches the steady state condition.

1 Introduction

Urban traffic congestion management with physics-based traffic coordination modeling has been extensively studied over the past three decades. A network of interconnected roads (NOIR) can be spatially discretized using the Cell Transmission Model (CTM) that applies the mass conservation principle to traffic coordination (1; 2). The Fundamental Diagram (3; 4) is frequently applied to model congestion with a flow-density relation at every traffic cell. While the Fundamental Diagram can successfully determine the traffic state for small-scale urban road networks, it may not properly analyze or control congestion in large traffic networks. Backward propagation, spill-back congestion, and shock-wave propagation are also difficult to accurately model.

1.1 Related Work

Physics-based control approaches have been previously proposed to deal with traffic coordination challenges. Fixed-cycle control is the traditional approach to operating traffic signals at intersections. For example, the traffic network study tool (5; 6) has been deployed to optimize traffic signal timing. Fuzzy control systems (7; 8) have also been proposed to optimize traffic light signal timings. The Fundamental Diagram is a popular physics-based approach to determine traffic state (flow-density relation) (3; 4) and to model dynamic traffic coordination (9). This diagram has also been used to describe spillback congestion (10; 11) and backward propagation (12; 13) effects into traffic simulation and to compute feasibility conditions for traffic congestion control.

Mass flow conservation (14) applies first order traffic dynamics, dynamic traffic assignment (15; 16), and a cell transmission model (1; 17) to model and control freeway traffic coordination. Model predictive control (MPC) is a common approach for model-based traffic flow optimization (18; 19; 20). As an example, MPC was used to determine the optimal platooning speed for automated highway systems (AHS) in (21). Fuzzy logic (22; 23; 24; 25), neural network (26; 27; 28; 29), Markov Decision Process (MDP) (30; 31), formal methods (32; 33), mixed nonlinear programming (MNLP) (34), and optimal control (14; 35) methods have all been applied to model-based traffic management.

This paper contributes a novel integrative behavioral-based physics-inspired approach to *obtain a microscopic data-driven traffic coordination model* and *resiliently control congestion in large-scale traffic networks*. The proposed modeling and control approach is formally presented and evaluated in traffic simulation studies. Following a summary of related work and a statement of contributions, preliminary notions of graph theory are first presented in Section 2. A problem statement is presented in Section 3. Section 4 models traffic coordination as a mass-conservation problem followed by a description of traffic congestion boundary control in Section 5. Simulation results presented in Section 6 are followed by concluding remarks in Section 7.

1.2 Contribution

This paper offers a new behavioral-based approach for control traffic coordination in a network of interconnected roads (NOIR). We model traffic coordination as a mass conservation problem governed by the continuity partial differential equation (PDE). By spatial and temporal discretization of traffic coordination, traffic dynamics is expressed by a probabilistic process controlled through the boundary road elements of the NOIR. The paper uses linear temporal logic to formally specify the feasibility conditions at NOIR road elements. Given traffic feasibility conditions, optimal boundary inflow is assigned as the solution of an adaptive model-predictive control (MPC) problem with parameters that are consistently learned based on the empirical traffic information. Therefore, the optimal boundary inflow is continuously assigned as the solution of a constrained quadratic programming problem and incorporated into planning.

2 Graph Theory Notions

A NOIR consists of a finite set of serially-connected road elements, where $i \in \mathcal{V}$ represents a unique road element. The set $\mathcal{V}$ can be partitioned as $\mathcal{V} = \mathcal{V}_{in} \cup \mathcal{V}_{out} \cup \mathcal{V}_I$, where $\mathcal{V}_{in} = \{1, \cdots, N_{in}\}$, $\mathcal{V}_{out} = \{N_{in}+1, \cdots, N_{out}\}$, and $\mathcal{V}_I = \{N_{out}+1, \cdots, N\}$ define index numbers of inlet, outlet, and interior road elements, respectively. Interactions between road elements are defined by graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$, with vertices $\mathcal{V}$ and edges $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$. For every road element $i \in \mathcal{V}$, the *in-neighbor set* $\mathcal{I}_i \triangleq \{j \mid (j,i) \in \mathcal{E}\} \subset \mathcal{V}_{in} \cup \mathcal{V}_I$ specifies upstream adjacent road elements, and the *out-neighbor set* $\mathcal{O}_i \triangleq \{j \mid (i,j) \in \mathcal{E}\} \subset \mathcal{V}_{out} \cup \mathcal{V}_I$ defines downstream adjacent road elements. Traffic enters $i \in \mathcal{V}$ from an in-neighbor node belonging to $\mathcal{I}_i$ and exits from $i \in \mathcal{V}$ toward an out-neighbor node belonging to $\mathcal{O}_i$.

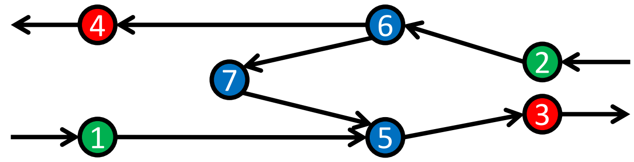

FIGURE 1. Simple NOIR example with 7 unidirectional roads, representing a simple roundabout

As an example, consider the example NOIR shown in Fig. 1 with 7 unidirectional roads. Every road element is identified by a unique index number $i \in \mathcal{V} = \{1, \cdots, 7\} = \mathcal{V}_{in} \bigcup \mathcal{V}_{out} \bigcup \mathcal{V}_I$, where $\mathcal{V}_{in} = \{1,2\}$, $\mathcal{V}_{out} = \{3,4\}$, and $\mathcal{V}_I = \{5,6,7\}$. The graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is defined by $\mathcal{V} = \{(1,5),(5,3),(2,6),(6,4),(6,7),(7,5)\}$.

Assumption. The paper assumes that $\mathcal{I}_i = \emptyset$ and $|\mathcal{O}_i| = 1$ for every inlet element $i \in \mathcal{V}_{in}$, where $|\cdot|$ is the set cardinality symbol. This paper also assumes that $\left|\mathcal{I}_j\right| = 1$ and $\mathcal{O}_j = \emptyset$, for every outlet element $j \in \mathcal{V}_{out}$.

3 Problem Statement

The traffic coordination control problem is defined by tuple M given by tuple $\mathrm{M} = (\mathbf{X}, \mathbf{U}, \mathcal{F}, \mathrm{C})$, where $\mathbf{X} \subset \mathbb{R}^{N-N_{out}}$. Vector $\mathbf{x} = \begin{bmatrix} \rho_{N_{out}+1} & \cdots & \rho_N \end{bmatrix}^T \in \mathbf{X}$ defines traffic density of interior road elements across the NOIR, where ρ_i is the traffic density at interior road element $i \in \mathcal{V}_I$. $\mathbf{U} \subset \mathbb{R}^{N_{in}}$, and vector $\mathbf{u} = \begin{bmatrix} u_1 & \cdots & u_{N_{in}} \end{bmatrix}^T \in \mathbf{U}$ defines the boundary inflow, where u_i is the inflow at inlet boundary element $i \in \mathcal{V}_{in}$. Because traffic density and boundary inflow are finite, $\mathbf{X} \subset \mathbb{R}^{N-N_{out}}$ and $\mathbf{U} \subset \mathbb{R}^{N_{in}}$ are compact. Furthermore, $\mathcal{F} : \mathbf{X} \times \mathbf{U} \to \mathbf{X}$ is the *traffic state transition function* defined as follows:

$$\mathcal{F}(\mathbf{x}, \mathbf{u}) = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \tag{1}$$

where constant matrices $\mathbf{A} \in \mathbb{R}^{(N-N_{out})\times(N-N_{out})}$ and $\mathbf{B} \in \mathbb{R}^{(N-N_{out})\times N_{in}}$ will be defined in Section 4 and $\mathbf{A} \in \mathbb{R}^{(N-N_{out})\times(N-N_{out})}$ is called the *tendency matrix*. Moreover, $\mathrm{C} : \mathbf{X} \times \mathbf{U} \to \mathbb{R}_{\geq 0}$ is the traffic coordination cost defined based on the traffic density distribution across the NOIR.

4 Traffic Coordination Modeling

The mass-conservation law is applied to obtain microscopic traffic dynamics across the NOIR. Therefore, traffic coordination at road element $i \in \mathcal{V}$ is given by

$$\rho_i[k+1] = \rho_i[k] + y_i[k] - z_i[k], \tag{2}$$

where $k \in \mathbb{N}$ denotes discrete time, $\rho_i[k]$ is the traffic density of road element $i \in \mathcal{V}$, and

$$y_i[k] = \begin{cases} u_i[k] & i \in \mathcal{V}_{in} \\ \sum_{j\in \mathcal{I}_i \bigcap \mathcal{V}_{in}} u_j[k] + \sum_{j\in \mathcal{I}_i \bigcap (\mathcal{V}\setminus\mathcal{V}_{in})} \bar{p}_j \bar{q}_{i,j}\rho_j[k] & i \in \mathcal{V}\setminus\mathcal{V}_{in} \end{cases} \tag{3a}$$

$$z_i[k] = \begin{cases} \bar{p}_i\rho_i[k] & i \in \mathcal{V}_I \\ y_i[k] & i \in \mathcal{V}\setminus\mathcal{V}_I \end{cases} \tag{3b}$$

are the traffic inflow and outflow, respectively, at road element $i \in \mathcal{V}$ over time interval $[t_k, t_{k+1}]$. Note that $u_i[k]$ can be controlled at inlet boundary element $i \in \mathcal{V}_{in}$, $\bar{p}_i \in [0,1]$ is the outflow probability of road element $i \in \mathcal{V}\setminus\mathcal{V}_{in}$, determining the fraction of cars leaving road element $i \in \mathcal{V}_I$ over time interval $t \in [t_k, t_{k+1}]$. Also, tendency probability $\bar{q}_{i,j}$ is the fraction of $z_j[k]$ directed from j toward $i \in \mathcal{O}_j$ over time interval $t \in [t_k, t_{k+1}]$, where $\sum_{i\in\mathcal{O}_j} \bar{q}_{i,j} = 1$. By convention, $\bar{q}_{i,j} = 0$ if $(j,i) \notin \mathcal{E}$.

Remark: Since $\mathcal{O}_j = \emptyset$ for $j \in \mathcal{V}_{out}$, traffic is not directed from an outlet boundary element towards an interior element or an inlet boundary element. Therefore for $j \in \mathcal{V}_{out}$, the quantities $\bar{q}_{1,j}$ through $\bar{q}_{N,j}$ are all zero. Therefore,

$$\forall i \in \mathcal{V}_I, \qquad y_i[k] = \sum_{j\in\mathcal{I}_i\bigcap\mathcal{V}_{in}} u_j[k] + \sum_{j\in\mathcal{I}_i\bigcap\mathcal{V}_I} \bar{p}_j\bar{q}_{i,j}\rho_j[k]. \tag{4}$$

4.1 Traffic state transition function

Per Eq. (3b), $y_i[k] = z_i[k]$ for every road element $i \in \mathcal{V}_{in}\bigcup\mathcal{V}_{out}$ at every discrete time k. Therefore, traffic density of every road element $i \in \mathcal{V}_{in}\bigcup\mathcal{V}_{out}$ remains constant but traffic density can change with time over the interior road elements. We define positive-definite and diagonal matrix

$$\mathbf{P} = \begin{bmatrix} \bar{p}_{N_{out}+1} & & 0 \\ & \ddots & \\ 0 & & \bar{p}_N \end{bmatrix} \in \mathbb{R}^{(N-N_{out})\times(N-N_{out})}. \tag{5}$$

We also define non-negative matrix $\mathbf{Q} = [Q_{ij}] = \left[\bar{q}_{i+N_{out},j+N_{out}}\right] \in \mathbb{R}^{(N-N_{out})\times(N-N_{out})}$ with ij entry $Q_{ij} = \bar{q}_{i+N_{out},j+N_{out}}$ specifying the tendency of traffic at interior node $j+N_{out} \in \mathcal{V}_I$ to move towards node $(i+N_{out}) \in \mathcal{O}_{j+N_{out}}$ at any time $t \in [t_k, t_{k+1}]$.

Traffic Tendency Matrix: Given **P** and **Q**, we define the *tendency matrix* $\mathbf{A} \in \mathbb{R}^{(N-N_{out})\times(N-N_{out})}$ as follows:

$$\mathbf{A} = \mathbf{I} - \mathbf{P} + \mathbf{QP}, \tag{6}$$

where $\mathbf{I} \in \mathbb{R}^{(N-N_{out})\times(N-N_{out})}$ is the identity matrix. Assuming traffic is updated by Eq. (2) at every road element $i \in \mathcal{V}_I$, density vector $\mathbf{x}[k] = \left[\rho_{N_{out}+1}[k] \ \cdots \ \rho_N[k]\right]^T$ is updated by the following network dynamics:

$$\mathbf{x}[k+1] = \mathcal{F}(\mathbf{x}[k], \mathbf{u}[k]) = \mathbf{A}\mathbf{x}[k] + \mathbf{B}\mathbf{u}[k], \tag{7}$$

where $\mathbf{u}[k] = \left[u_1[k] \ \cdots \ u_{N_{in}}[k]\right]^T$ is the boundary inflow vector, $\mathbf{B} = \left[B_{ij}\right] \in \mathbb{R}^{(N-N_{out})\times N_{in}}$, and B_{ij} is a constant matrix defined as $B_{ij} = \begin{cases} 1 & j \in \mathcal{I}_{i+N_{out}} \\ 0 & \text{otherwise} \end{cases}$.

Theorem 1. *The traffic state transition, defined by dynamics* (7)*, is BIBO stable when the following conditions are satisfied:*

1. *There exists at least one directed path from every inlet boundary road element toward the interior of road element $i \in \mathcal{V}_I$.*
2. *There exists at least one directed path from the interior of road element $i \in \mathcal{V}_I$ toward every outlet boundary road element.*

Proof: Because there exists a path from each boundary node to every interior node of graph $\mathcal{G}$, non-negative matrix **A** is irreducible, and the sum of the column entries of **A** is one or less than one. Entries of column i of matrix **A** sum to 1 if no out-neighbors of road element $i+N_{out}$ are outlet boundary nodes, i.e. $\mathcal{O}_{i+N_{out}}\bigcap\mathcal{V}_{out} = \emptyset$. Otherwise, the sum of the entries of column i of matrix **A** is 0 or a positive number between 0 and 1. Consequently, the spectral radius of **A** is less than 1. When $\mathbf{x}[k]$ is updated by discrete traffic dynamics (7), we can write

$$\mathbf{x}[k+1] = \left[\mathbf{\Gamma}_k \ \cdots \ \mathbf{\Gamma}_1 \ \mathbf{I}\right] \begin{bmatrix} \mathbf{x}[1] \\ \mathbf{Bu}[1] \\ \vdots \\ \mathbf{Bu}[k] \end{bmatrix}. \tag{8}$$

Eigenvalues of matrix **A** are all placed inside a unit disk centered at the origin. Because $\mathbf{x}[1]$ is finite and $\mathbf{u}[k]$ is bounded at every discrete time k, there exists a $z_{max} < \infty$ such that $\mathbf{x}[1] <$

$z_{max}\mathbf{1}_{N-N_{out},1}$, and $\mathbf{B}\mathbf{x}[k] < z_{max}\mathbf{1}_{N-N_{out},1}$ for $k = 1,2,\cdots$. Thus

$$\begin{aligned}\mathbf{x}^T[k+1]\mathbf{x}[k+1] &\le z_{max}^2\mathbf{1}^T\left(\mathbf{I}+\sum_{h=1}^{k}\mathbf{\Psi}_h^T\mathbf{\Psi}_h\right)\mathbf{1}\\ &\le z_{max}^2(N-N_{out})\left(\sum_{h=0}^{k}r^h\right)\le\frac{z_{max}^2(N-N_{out})}{1-r},\end{aligned} \tag{9}$$

where r is the spectral radius of matrix $\mathbf{\Psi}_h^T\mathbf{\Psi}_h$ for $h = 1,2,\cdots$. Because $r < 1$, the right-hand side of Eq. (9) is bounded and the BIBO stability of traffic dynamics (7) is proven.

4.2 Traffic Feasibility Conditions

Linear Temporal Logic (LTL) is used to specify the feasibility conditions of the conservation-based traffic coordination dynamics given in equation (7) (36). Every LTL formula consists of a set of atomic propositions, logical operators, and temporal operators. Logical operators include $\neg$ ("negation"), $\vee$ ("disjunction"), $\wedge$ ("conjunction"), and $\Rightarrow$ ("implication"). LTL formulae also use temporal operators $\square$ ("always"), $\bigcirc$ ("next"), $\Diamond$ ("eventually"), and $\mathcal{U}$ ("until").

We extend discrete-time LTL with the syntactic sugar $\square_{\{0,\ldots,N_\tau\}}$ whose intuition is that the state M = $(\mathbf{X},\mathbf{U},\mathcal{F},\mathrm{C})$ satisfies $\square_{\{0,\ldots,N_\tau\}}\varphi$ at time k if and only if it satisfies φ from time $k+0 = k$ to time $k+N_\tau$, included. In discrete-time LTL, the operator $\square_{\{0,\ldots,N_\tau\}}$ can be defined syntactically as:

$$\square_{\{0,\ldots,N_\tau\}}\varphi \triangleq \varphi\wedge\bigcirc\varphi\wedge\bigcirc\bigcirc\varphi\wedge\ldots\wedge\underbrace{\bigcirc\cdots\bigcirc}_{N_\tau \text{ times}}\varphi$$

The notation is inspired by Metric Temporal Logic MTL (37) and Signal Temporal Logic STL (38), which feature a similar operator in continuous time.

Four traffic feasibility conditions are formally specified below to serve as formal constraints for the subsequent optimal control definition. In particular, these feasibility conditions are used to determine admissible boundary inflow $\mathbf{u}[k]\in\mathbf{U}$ at every discrete time k.

Feasibility Condition 1: Traffic density, defined as the number of cars at a road element, is a positive quantity everywhere in the NOIR. Also, it is assumed that every road element has maximum capacity $\rho_{\max}$. Therefore, the number of cars cannot exceed $\rho_{\max}$ in every road element $i\in\mathcal{V}_I$. These two requirements can be formally specified as follows:

$$\boxed{\bigwedge_{i\in\mathcal{V}}\square_{\{0,\ldots,N_\tau\}}(\rho_i\ge 0\ \wedge\ \rho_i\le\rho_{\max})}. \tag{Φ_1}$$

If feasibility condition Φ_1 is satisfied at every road element, then, traffic over-saturation is avoided everywhere in the NOIR at every discrete times k.

Feasibility Condition 2: Fraction $\bar{q}_{j,i}$ of the outflow z_i directed from $i\in\mathcal{V}$ toward $j\in O_i$ must not exceed the available capacity of road element i denoted by $C_j[k] = \rho_{\max}-\rho_j[k]$ at every discrete time k. This condition can be formally specified by

$$\boxed{\bigwedge_{i\in\mathcal{V}}\bigwedge_{j\in O_i}\square_{\{0,\ldots,N_\tau\}}\left(\bar{q}_{j,i}z_i\le C_j\right)}. \tag{Φ_2}$$

Feasibility Condition 3: The inflow y_i must not exceed the available available capacity $C_i[k]$ at every discrete times k. This requirement is formally specified by the following LTL formula:

$$\boxed{\bigwedge_{i\in\mathcal{V}}\square_{\{0,\ldots,N_\tau\}}(y_i\le C_i)}. \tag{Φ_3}$$

Feasibility Condition 4: The boundary inflow needs to satisfy the following feasibility condition at every discrete time k:

$$\boxed{\square_{\{0,\ldots,N_\tau\}}(\mathbf{u}\in\mathbf{U})}. \tag{Φ_4}$$

While Feasibility Conditions 1 through 4 need to be satisfied at every discrete time k, the following "optional" condition is also implemented when inflow demand is high:

Optional Condition 5: Boundary inflow should satisfy the following feasibility condition at every discrete time k:

$$\boxed{\square_{\{0,\ldots,N_\tau\}}\left(\sum_{i\in\mathcal{V}_{in}}u_i = u_0\right)}. \tag{Φ_5}$$

Boundary condition (Φ_5) constrains the number of vehicles entering the NOIR to be exactly u_0 at any time k. Note that u_0 is an upper bound on vehicles entering the NOIR. However, in the simulation results presented, traffic demand is significant such that the NOIR is maximally utilized by as many vehicles as possible.

5 Traffic Coordination Control

The boundary inflow $\mathbf{u}[k]$ can be controlled by ramp meters situated at the inlet boundary nodes at every discrete time k. Ramp meters apply an MPC control design to determine the optimal boundary inflow $\mathbf{u}[k]$ so that the traffic congestion can be effectively and resiliently managed.

The optimal boundary inflow $\mathbf{u}[k] = \mathbf{u}^*$ is determined by minimizing the N_τ-step expected cost

$$\mathrm{C} = \sum_{\tau=1}^{N_\tau}\left(\mathbf{x}^T[k+\tau]\mathbf{x}^T[k+\tau]+\beta\mathbf{u}^T[k+\tau]\mathbf{u}[k+\tau]\right), \tag{10}$$

where $C = C(\mathbf{x}[k], \mathbf{u}[k+1], \cdots, \mathbf{u}[k+N_\tau])$ scaling parameter $\beta > 0$ is constant and all traffic feasibility conditions must be satisfied. The optimal control $\mathbf{u}[k] = \mathbf{u}^*$ is assigned as the solution of a constrained quadratic programming problem that can be formally specified as follows:

$$(\mathbf{u}[k+1], \cdots, \mathbf{u}[k+N_\tau]) = \underset{(\mathbf{u}[k+1], \cdots, \mathbf{u}[k+N_\tau]) \in \mathbf{U}^{N_\tau}}{\text{argmin}} \quad C \tag{11}$$

subject to constraints (Φ_1), (Φ_2), (Φ_3), (Φ_4) and (Φ_5). Cost function C can only be defined based on $\mathbf{x}[k], \mathbf{u}[k+1], \cdots, \mathbf{u}[k+N_\tau]$ at every discrete time k. Therefore, $\mathbf{u}^* = \mathbf{u}[k]$ can be assigned as the solution of a constrained quadratic programming problem at every discrete time k.

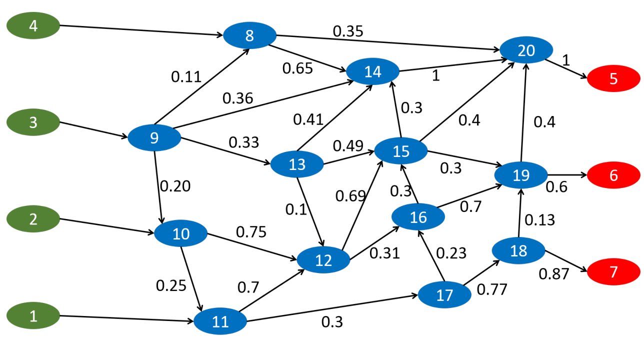

FIGURE 2. Communication graph representing an example NOIR .

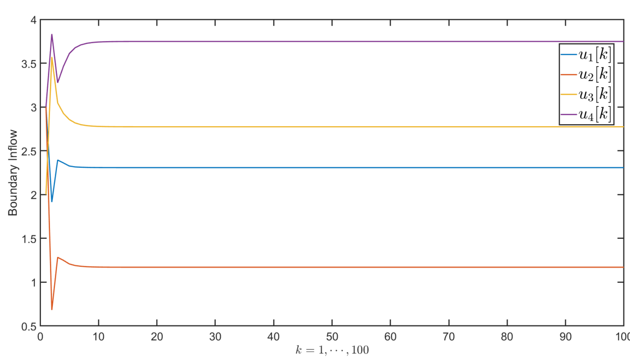

FIGURE 3. Boundary control input $u_1[k]$ through $u_8[k]$ for $k = 1, \cdots, 100$

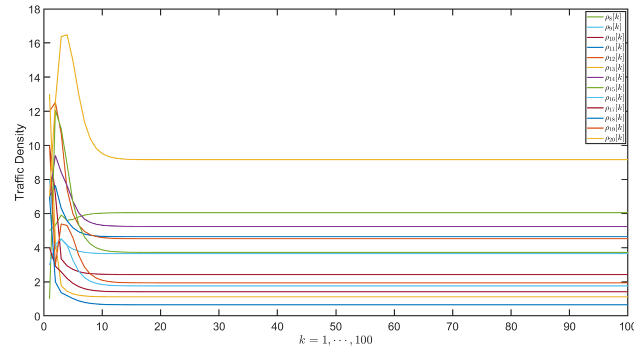

FIGURE 4. Traffic density at interior road elements for $k = 1, \cdots, 100$.

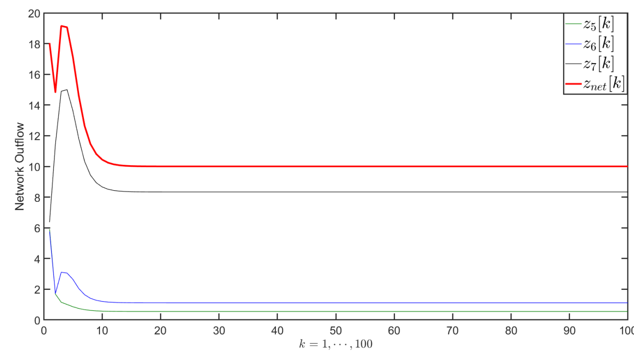

FIGURE 5. Traffic outflow at outlet boundary elements 5, 6, and 7 and network traffic outflow z_{net} for $k = 1, \cdots, 100$.

6 Simulation Results

Consider the NOIR consists of 20 unidirectional roads (road elements) shown in Fig. 2. Road elements are defined by set $\mathcal{V} = \{1, \cdots, 20\}$ where $\mathcal{V} = \mathcal{V}_{in} = \bigcup \mathcal{V}_{out} \bigcup \mathcal{V}_I$ where $\mathcal{V}_{in} = \{1, \cdots, 4\}$, $\mathcal{V}_{out} = \{5, 6, 7\}$, $\mathcal{V}_I = \{8, \cdots, 20\}$, $N_{in} = |\mathcal{V}_{in}| = 4$, $N_{out} = |\mathcal{V}_{out}| = 3$, and $N_I = |\mathcal{V}_I| = 13$. For simulation, $\bar{p}_8 = 0.67$, $\bar{p}_9 = 0.76$, $\bar{p}_{10} = 0.71$, $\bar{p}_{11} = 0.59$, $\bar{p}_{12} = 0.73$, $\bar{p}_{13} = 0.82$, $\bar{p}_{14} = 0.94$, $\bar{p}_{10} = 0.83$, $\bar{p}_{16} = 0.69$, $\bar{p}_{17} = 0.58$, $\bar{p}_{10} = 0.97$, $\bar{p}_{19} = 0.96$ and $\bar{p}_{20} = 0.91$ are the average outflow probabilities at interior road elements 8 through 20. Furthermore, the traffic tendency proabilities shown in Fig. 2 are used to set up matrix $\mathbf{Q}$. Given $\mathbf{P}$ and $\mathbf{Q}$, matrix $\mathbf{A} \in \mathbb{R}^{13 \times 13}$ is computed using (6).

We assume that $u_0 = 10$, thus the number of vehicles entering the NOIR are restricted to be 10 at any time k. Also, $\rho_{\max} = 40$ is selected for the simulation. Boundary control inflows u_1 through u_4 are plotted versus discrete time k for $k = 1, \cdots, 100$ in Fig. 3. Fig. 4 plots the traffic density in all road elements versus discrete time k. Figs. 3 and 4 imply that traffic density reaches the steady state values after about 15 time steps in simulation while traffic consistently enters the NOIR through the inlet boundary nodes.

Furthermore, traffic outflows z_5, z_6, z_7 are plotted versus discrete time k in Fig. 5. It is observed that

$$\forall k > 15, \qquad z_{net}[k] = z_5[k] + z_6[k] + z_7[k] \cong 10$$

This implies that the net traffic inflow ($u_0 = 10$) is approximately the same as the net traffic outflow for $k > 15$.

7 Conclusion

This paper offers a behavioral-based physics-inspired approach to effectively model and control traffic congestion. This paper proposes learning traffic flow-density relations using empirical traffic data rather than the traditional Fundamental Diagram. This approach offers several benefits: (i) Traffic data is consistently incorporated into the model, (ii) Microscopic properties of a traffic system are incorporated into planning, and (iii) Resilience of traffic congestion control is improved. Furthermore, feasibility conditions for traffic coordination in a large-scale urban network are formally specified using liner temporal logic. Simulation studies show reasonable steady-state traffic flow properties. Future work with real traffic data will be required to learn realistic traffic tendencies for specific real-world road networks.

ACKNOWLEDGMENT

This work has been supported by the National Science Foundation under Award Nos. 1914581 and 1739525.

References

[1] Daganzo, C. F., 1995. "The cell transmission model, part ii: network traffic". *Transportation Research Part B: Methodological,* ***29***(2), pp. 79–93.

[2] Gomes, G., and Horowitz, R., 2006. "Optimal freeway ramp metering using the asymmetric cell transmission model". *Transportation Research Part C: Emerging Technologies,* ***14***(4), pp. 244–262.

[3] Zhang, J., Klingsch, W., Schadschneider, A., and Seyfried, A., 2012. "Ordering in bidirectional pedestrian flows and its influence on the fundamental diagram". *Journal of Statistical Mechanics: Theory and Experiment,* ***2012***(02), p. P02002.

[4] Zhang, J., Klingsch, W., Schadschneider, A., and Seyfried, A., 2011. "Transitions in pedestrian fundamental diagrams of straight corridors and t-junctions". *Journal of Statistical Mechanics: Theory and Experiment,* ***2011***(06), p. P06004.

[5] Robertson, D. I., 1969. *TRANSYT: a traffic network study tool*. National Academy of Science.

[6] Tiwari, G., Fazio, J., Gaurav, S., and Chatteerjee, N., 2008. "Continuity equation validation for nonhomogeneous traffic". *Journal of Transportation Engineering,* ***134***(3), pp. 118–127.

[7] Balaji, P., and Srinivasan, D., 2011. "Type-2 fuzzy logic based urban traffic management". *Engineering Applications of Artificial Intelligence,* ***24***(1), pp. 12–22.

[8] Chiu, S., 1992. "Adaptive traffic signal control using fuzzy logic". In Proceedings of the Intelligent Vehicles92 Symposium, IEEE, pp. 98–107.

[9] Han, K., Piccoli, B., Friesz, T. L., and Yao, T., 2012. "A continuous-time link-based kinematic wave model for dynamic traffic networks". *arXiv preprint arXiv:1208.5141,* ***4***, p. 35.

[10] Gentile, G., Meschini, L., and Papola, N., 2007. "Spillback congestion in dynamic traffic assignment: a macroscopic flow model with time-varying bottlenecks". *Transportation Research Part B: Methodological,* ***41***(10), pp. 1114–1138.

[11] Adamo, V., Astarita, V., Florian, M., Mahut, M., and Wu, J., 1999. "Modelling the spill-back of congestion in link based dynamic network loading models: a simulation model with application". In 14th International Symposium on Transportation and Traffic TheoryTransportation Research Institute.

[12] Gentile, G., 2015. "Using the general link transmission model in a dynamic traffic assignment to simulate congestion on urban networks". *Transportation Research Procedia,* ***5***, pp. 66–81.

[13] Long, J., Gao, Z., Ren, H., and Lian, A., 2008. "Urban traffic congestion propagation and bottleneck identification". *Science in China Series F: Information Sciences,* ***51***(7), p. 948.

[14] Jafari, S., and Savla, K., 2018. "On structural properties of feedback optimal control of traffic flow under the cell transmission model". *arXiv preprint arXiv:1805.11271*.

[15] Peeta, S., and Ziliaskopoulos, A. K., 2001. "Foundations of dynamic traffic assignment: The past, the present and the future". *Networks and spatial economics,* ***1***(3-4), pp. 233–265.

[16] Janson, B. N., 1991. "Dynamic traffic assignment for urban road networks". *Transportation Research Part B: Methodological,* ***25***(2-3), pp. 143–161.

[17] Daganzo, C. F., 1994. "The cell transmission model: A dynamic representation of highway traffic consistent with the hydrodynamic theory". *Transportation Research Part B: Methodological,* ***28***(4), pp. 269–287.

[18] Lin, S., De Schutter, B., Xi, Y., and Hellendoorn, H., 2012. "Efficient network-wide model-based predictive control for urban traffic networks". *Transportation Research Part C: Emerging Technologies,* ***24***, pp. 122–140.

[19] Jamshidnejad, A., Papamichail, I., Papageorgiou, M., and De Schutter, B., 2018. "Sustainable model-predictive control in urban traffic networks: Efficient solution based on general smoothening methods". *IEEE Transactions on Control Systems Technology,* ***26***(3), pp. 813–827.

[20] Tettamanti, T., Luspay, T., Kulcsar, B., Péni, T., and Varga, I., 2014. "Robust control for urban road traffic networks". *IEEE Transactions on Intelligent Transportation Systems,* ***15***(1), pp. 385–398.

[21] Baskar, L. D., De Schutter, B., and Hellendoorn, H., 2012. "Traffic management for automated highway systems using model-based predictive control". *IEEE Transactions on Intelligent Transportation Systems,* ***13***(2), pp. 838–847.

[22] Kammoun, H. M., Kallel, I., Casillas, J., Abraham, A., and Alimi, A. M., 2014. "Adapt-traf: An adaptive multiagent road traffic management system based on hybrid ant-hierarchical fuzzy model". *Transportation Research Part C: Emerging Technologies,* ***42***, pp. 147–167.

[23] Collotta, M., Bello, L. L., and Pau, G., 2015. "A novel approach for dynamic traffic lights management based on wireless sensor networks and multiple fuzzy logic controllers". *Expert Systems with Applications,* ***42***(13), pp. 5403–5415.

[24] Pau, G., Campisi, T., Canale, A., Severino, A., Collotta, M., and Tesoriere, G., 2018. "Smart pedestrian crossing management at traffic light junctions through a fuzzy-based approach". *Future Internet,* ***10***(2), p. 15.

[25] Yusupbekov, N., Marakhimov, A., Igamberdiev, H., and Umarov, S. X., 2016. "An adaptive fuzzy-logic traffic control system in conditions of saturated transport stream". *The Scientific World Journal,* ***2016***.

[26] Moretti, F., Pizzuti, S., Panzieri, S., and Annunziato, M., 2015. "Urban traffic flow forecasting through statistical and neural network bagging ensemble hybrid modeling". *Neurocomputing,* ***167***, pp. 3–7.

[27] Tang, J., Liu, F., Zou, Y., Zhang, W., and Wang, Y., 2017. "An improved fuzzy neural network for traffic speed prediction considering peri-

odic characteristic". *IEEE Transactions on Intelligent Transportation Systems,* ***18***(9), pp. 2340–2350.

[28] Akhter, S., Rahman, R., and Islam, A., 2016. "Neural network (nn) based route weight computation for bi-directional traffic management system". *International Journal of Applied Evolutionary Computation (IJAEC),* ***7***(4), pp. 45–59.

[29] Kumar, K., Parida, M., and Katiyar, V. K., 2015. "Short term traffic flow prediction in heterogeneous condition using artificial neural network". *Transport,* ***30***(4), pp. 397–405.

[30] Ong, H. Y., and Kochenderfer, M. J., 2016. "Markov decision process-based distributed conflict resolution for drone air traffic management". *Journal of Guidance, Control, and Dynamics*, pp. 69–80.

[31] Haijema, R., and van der Wal, J., 2008. "An mdp decomposition approach for traffic control at isolated signalized intersections". *Probability in the Engineering and Informational Sciences,* ***22***(4), pp. 587–602.

[32] Coogan, S., Arcak, M., and Belta, C., 2017. "Formal methods for control of traffic flow: Automated control synthesis from finite-state transition models". *IEEE Control Systems Magazine,* ***37***(2), pp. 109–128.

[33] Coogan, S., Gol, E. A., Arcak, M., and Belta, C., 2015. "Traffic network control from temporal logic specifications". *IEEE Transactions on Control of Network Systems,* ***3***(2), pp. 162–172.

[34] Christofa, E., Papamichail, I., and Skabardonis, A., 2013. "Person-based traffic responsive signal control optimization". *IEEE Transactions on Intelligent Transportation Systems,* ***14***(3), pp. 1278–1289.

[35] Wang, Y., Szeto, W., Han, K., and Friesz, T. L., 2018. "Dynamic traffic assignment: A review of the methodological advances for environmentally sustainable road transportation applications". *Transportation Research Part B: Methodological,* ***111***, pp. 370–394.

[36] Wongpiromsarn, T., Topcu, U., and Murray, R. M., 2009. "Receding horizon temporal logic planning for dynamical systems". In Proceedings of the 48h IEEE Conference on Decision and Control (CDC) held jointly with 2009 28th Chinese Control Conference, IEEE, pp. 5997–6004.

[37] Koymans, R., 1990. "Specifying real-time properties with metric temporal logic". *Real-time systems,* ***2***(4), pp. 255–299.

[38] Maler, O., and Nickovic, D., 2004. "Monitoring temporal properties of continuous signals". In *Formal Techniques, Modelling and Analysis of Timed and Fault-Tolerant Systems*. Springer, pp. 152–166.