The 15 Puzzle and Homological Stability in the Space Direction

Nicholas Wawrykow

Based on joint work with Jesús González and Matthew Kahle

The 15 Puzzle

Goal: Slide 15 labeled unit squares around a 4 by 4 rectangle to move from an initial configuration to a target configuration.

Initial Configuration

2	15	12	7
10	3	8	14
6	4	11	13
9	1	5	

Target Configuration

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

The 15 Puzzle: A Prize

Prize (Loyd 1896)

$1,000 to solve this puzzle:

1	2	3	4
5	6	7	8
9	10	11	12
13	15	14	

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

The 15 Puzzle: An Unwinnable Prize

Theorem (Johnson–Story 1879)

You can't. These configurations are in different components of the configuration space.

1	2	3	4
5	6	7	8
9	10	11	12
13	15	14	

1	2	3	4
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Proof Sketch

Theorem (Johnson–Story 1879)

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Configurations $\leftrightarrow$ elements of S_{15}

1	2	3	4
5	6	7	8
9	10	11	12
13	15	14	

$(14\,15)$

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5	6	7	8
9	10	11	12
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$(\,)$

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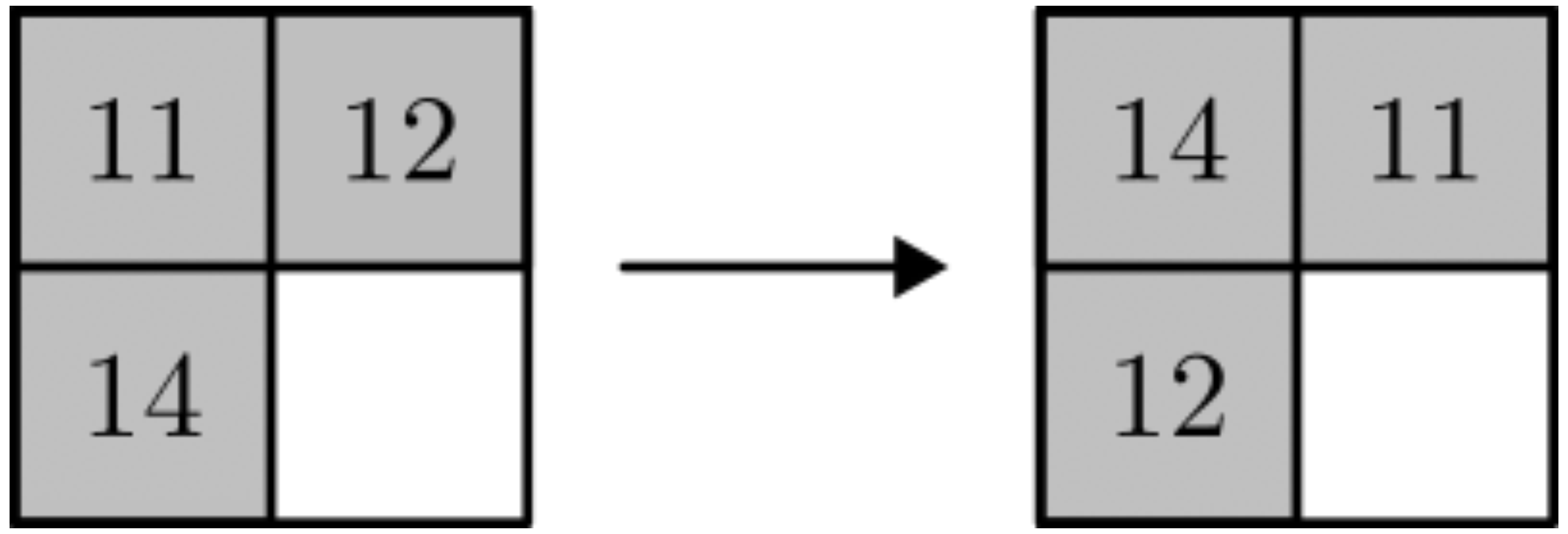
Up to rearrangement, can only make 3-cycles



$(11\,12\,14)$

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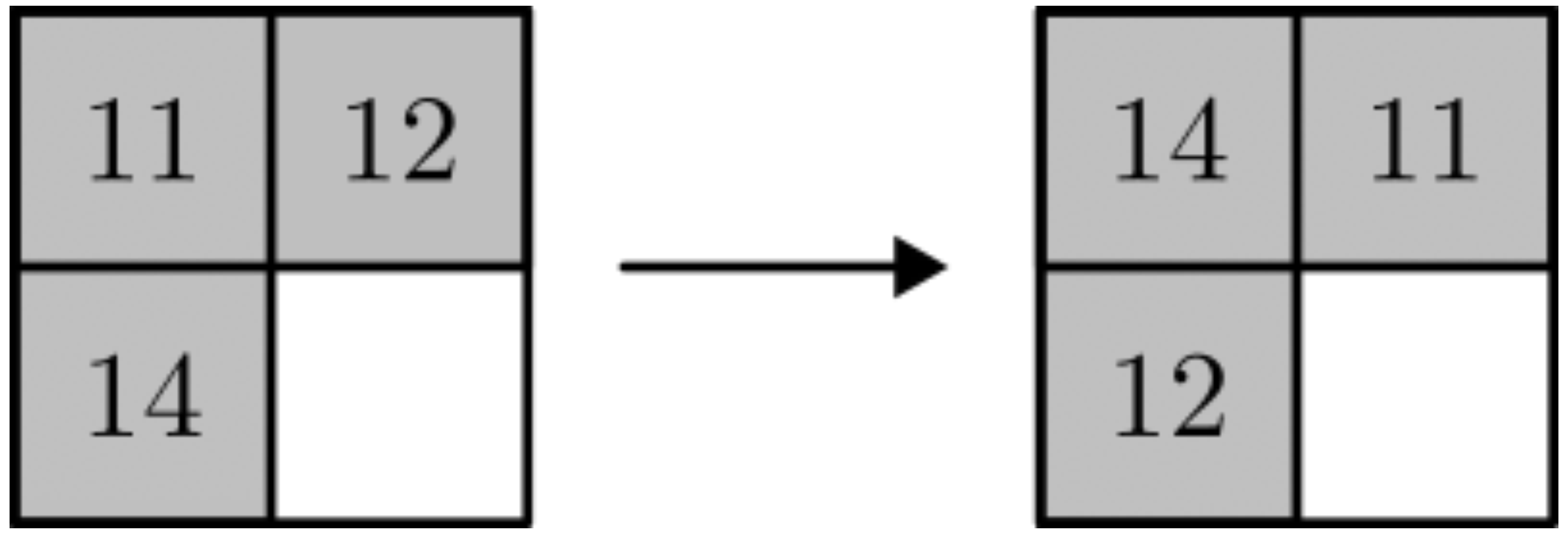

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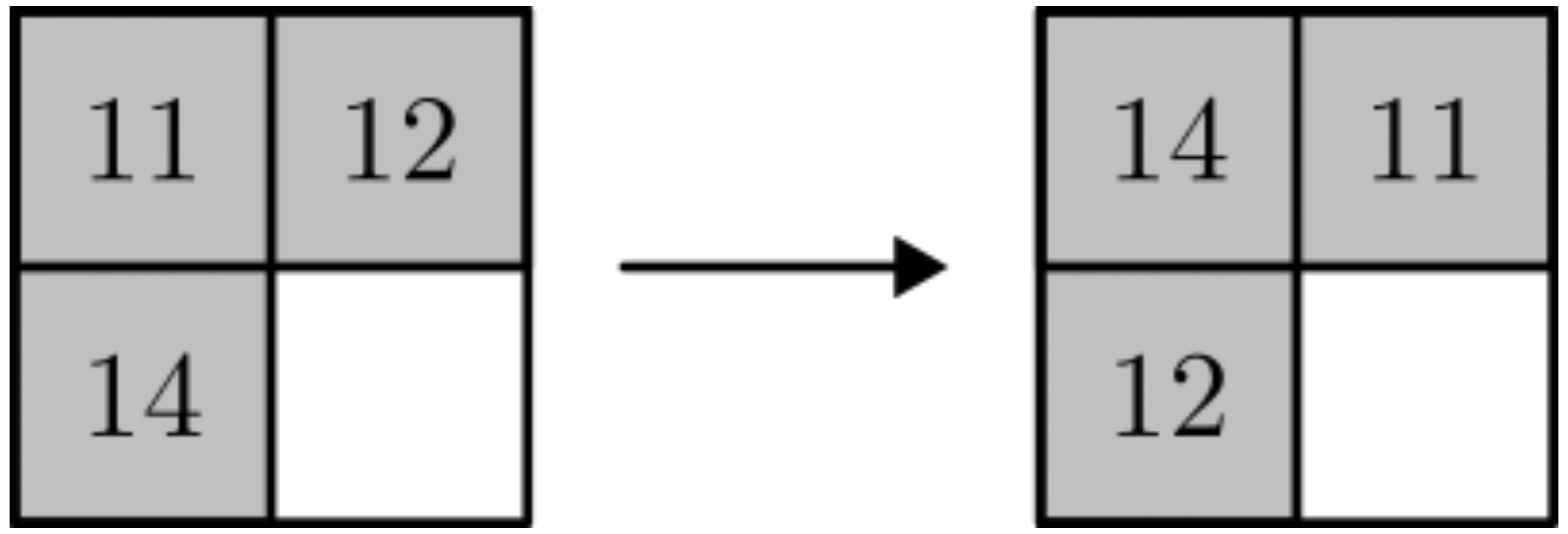


$(11\,12\,14)$

3-cycles generate A_{15}

Configurations differ by a transposition $\implies$ different cosets of S_{15}/A_{15}

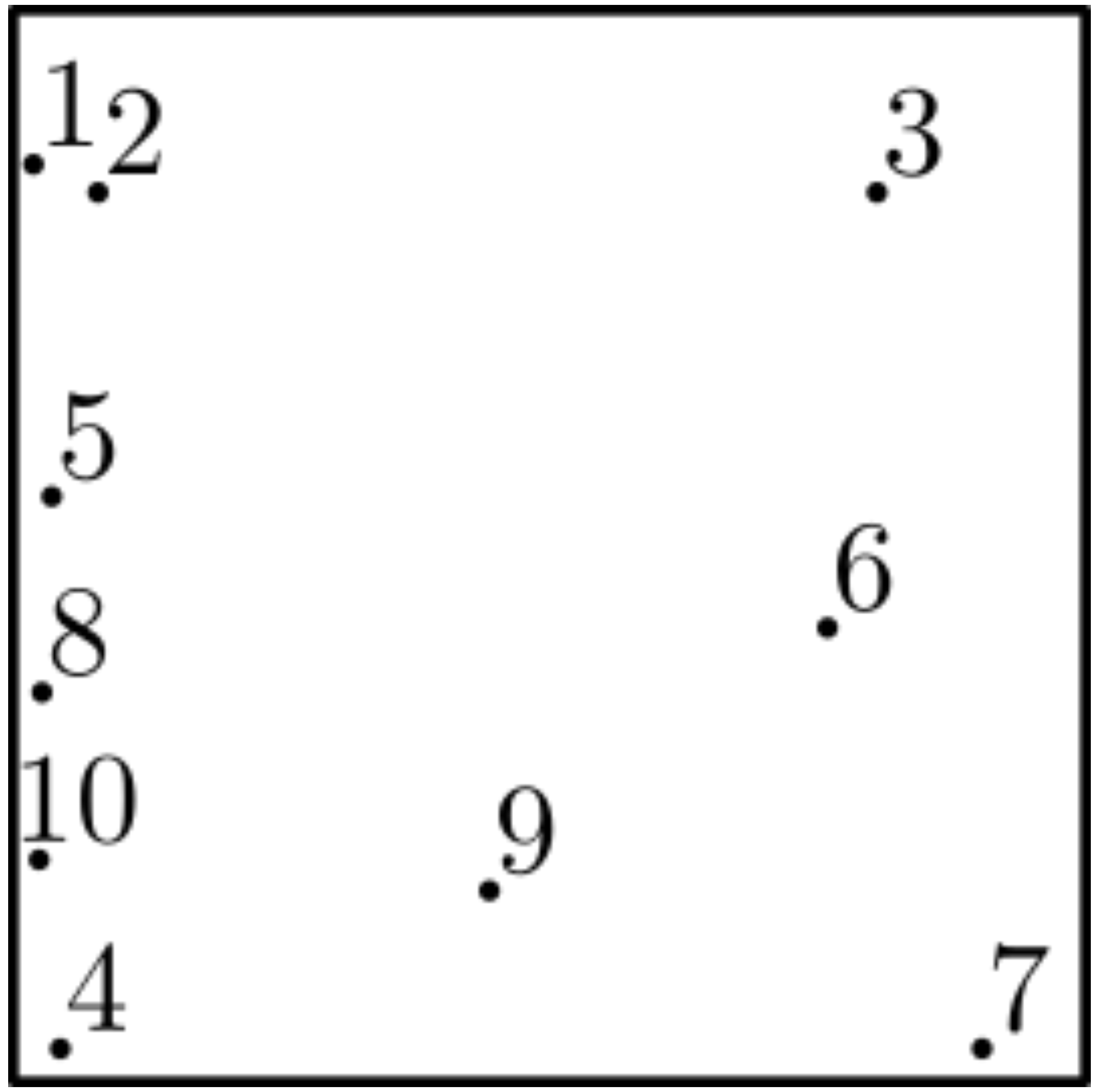
$(14\,15)$ $(\,)$

Configuration Space

Definition

Let X be a space. The *ordered configuration space of* n *points in* X is

$$F_n(X) := \{(x_1, \ldots, x_n) \in X^n | x_i \neq x_j \text{ for } i \neq j\}$$



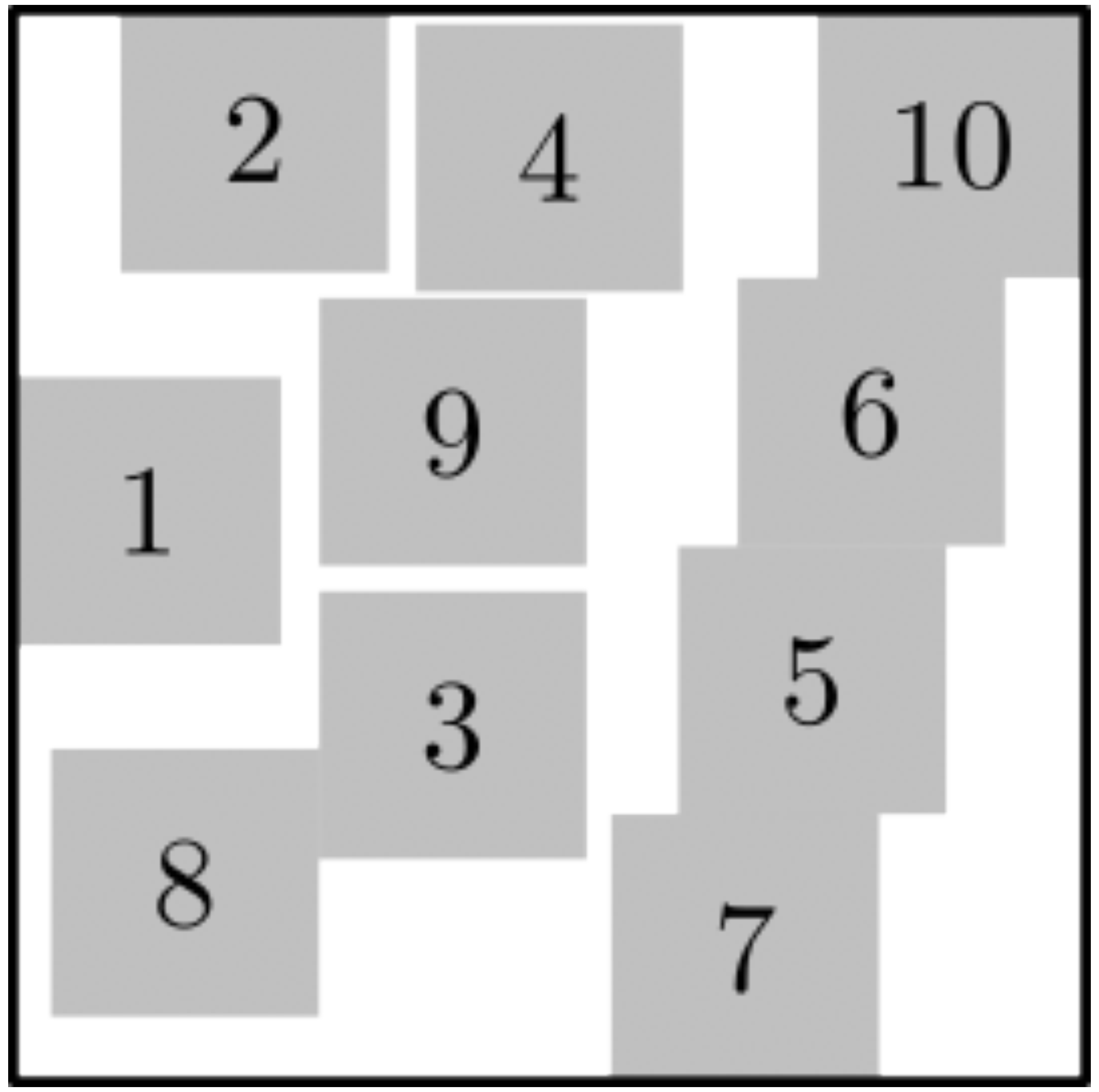
A configuration in $F_{10}(\mathbb{C})$

Square Configuration Space

Definition

The *ordered configuration space of n open unit squares in the w by h rectangle* is

$$SF_n(R_{w,h}) := \left\{ (z_1, \ldots, z_n) \in \mathbb{C}^n \,\middle|\, \begin{array}{c} \max\{|\mathrm{Re}(z_i)-\mathrm{Re}(z_j)|,|\mathrm{Im}(z_i)-\mathrm{Im}(z_j)|\}\geq 1, \\ \frac{1}{2}\leq \mathrm{Re}(z_i)\leq w-\frac{1}{2}, \frac{1}{2}\leq \mathrm{Im}(z_i)\leq h-\frac{1}{2} \end{array} \right\}.$$



A configuration in $SF_{10}(R_{4,4})$

Disk Configuration Space

Definition

Let (X, g) be a metric space. The *ordered configuration space of* n *open unit disks in* X is

$$DF_n(X) := \left\{ (x_1, \ldots, x_n) \in X^n \middle| \substack{d_g(x_i, x_j) \geq 1 \\ \text{and a "boundary" condition}} \right\}.$$

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Chemistry, statistical mechanics, and physics

Robotics and motion planning

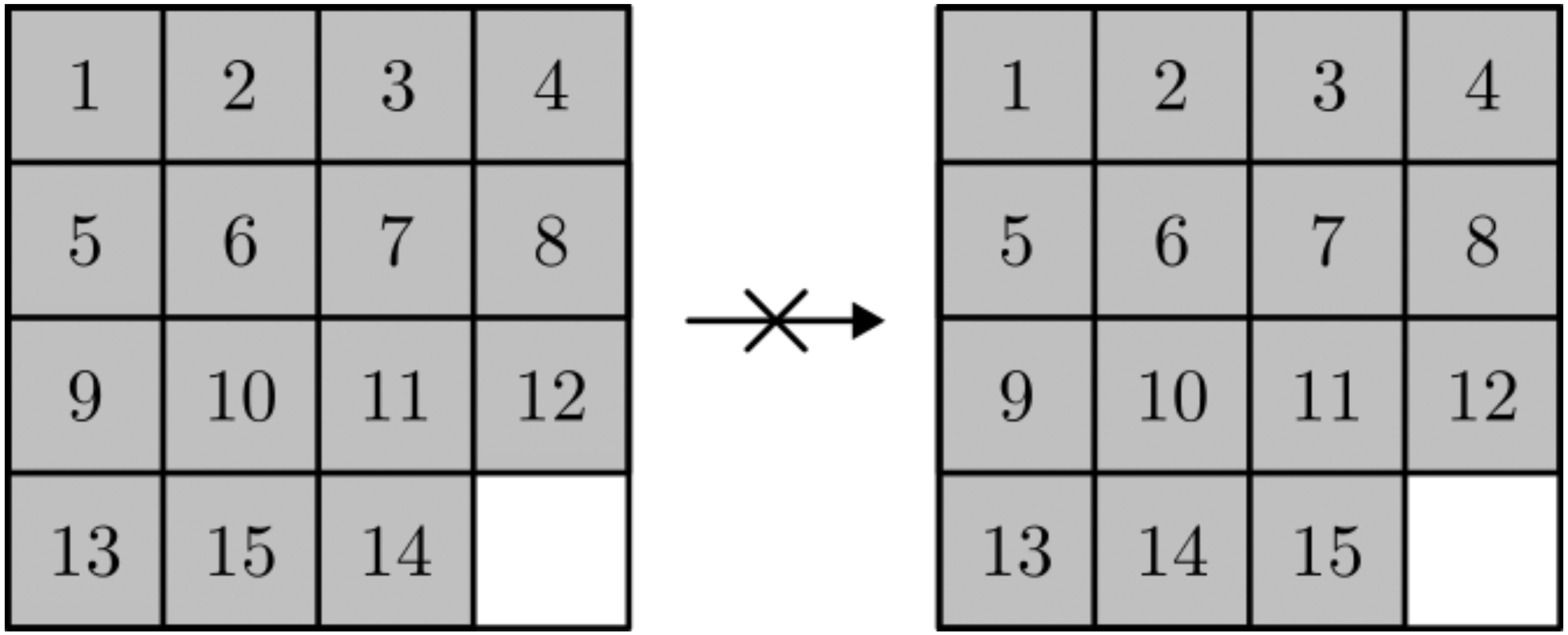
Approximations to $F_n(X)$ and interpolating between E_1 and E_2 algebras

Realizing braids

Making the Puzzle Solvable

Question

How many squares do you need to remove to ensure that the puzzle is solvable?



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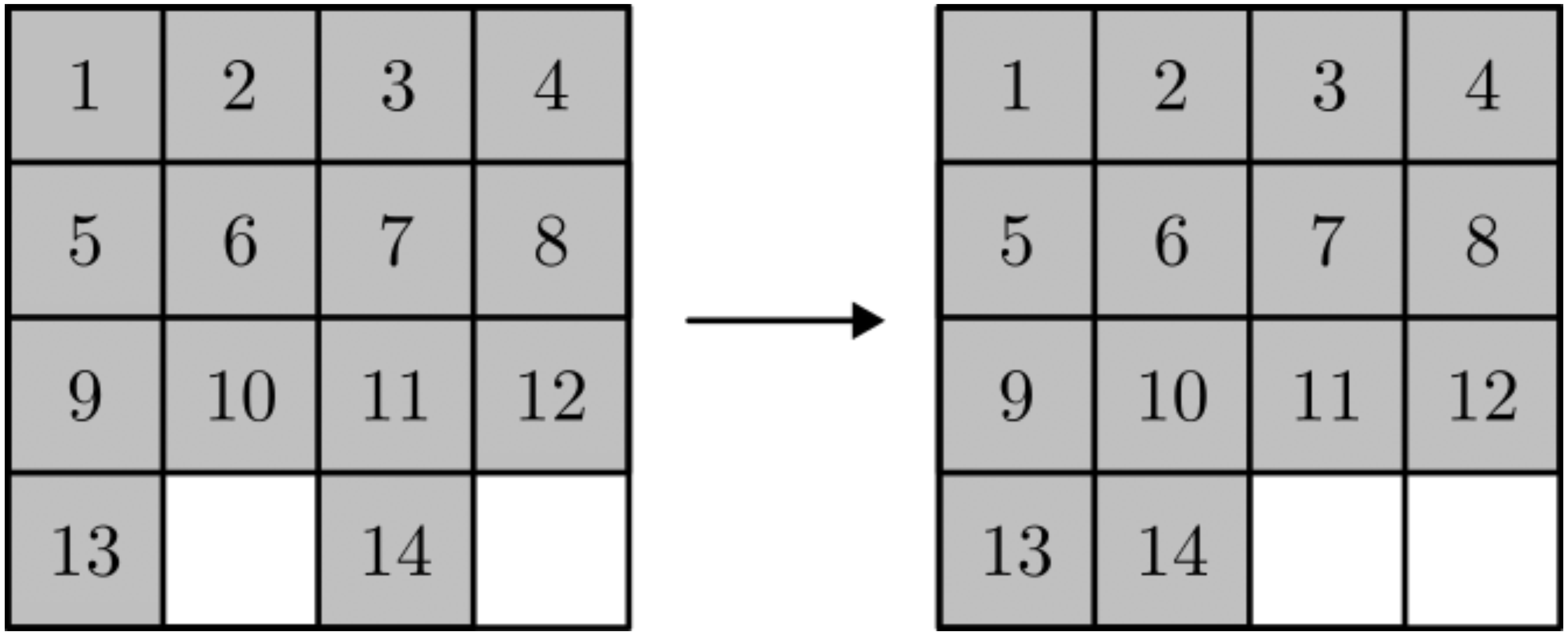

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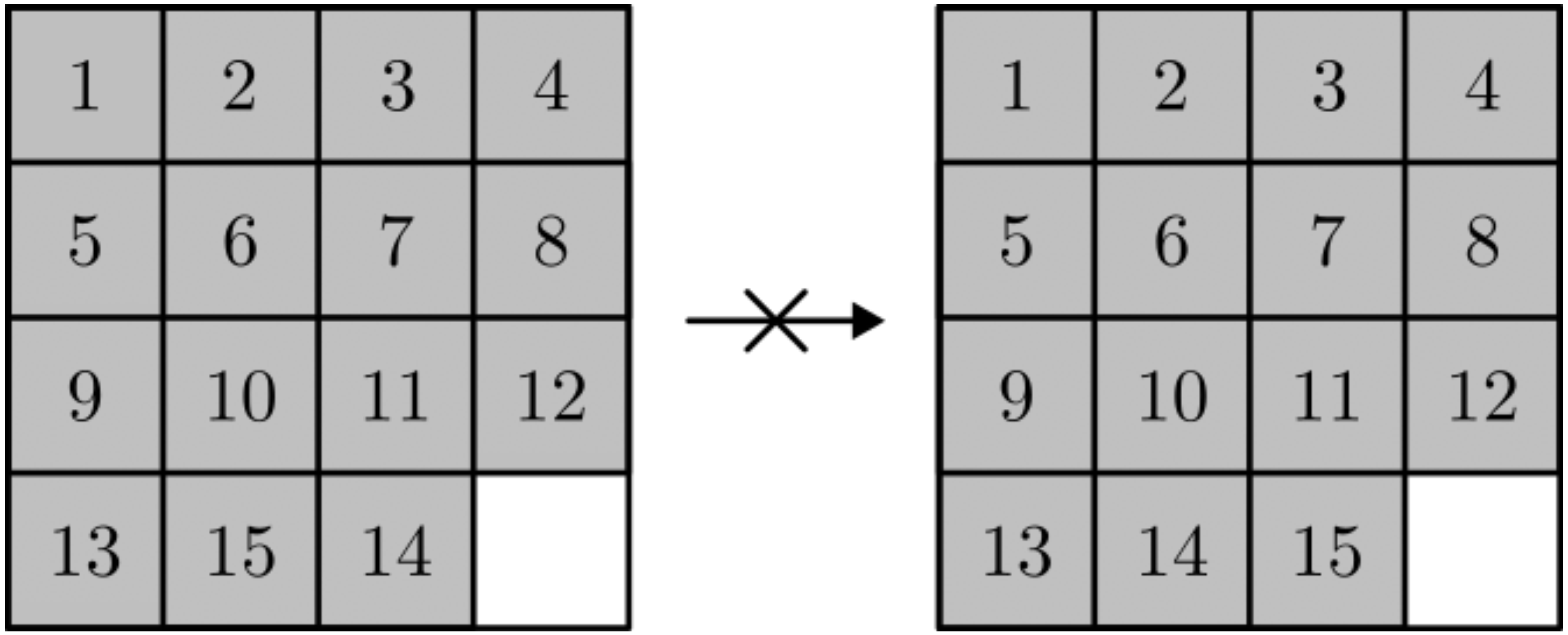
How big does the rectangle need to be to ensure that the puzzle is solvable?



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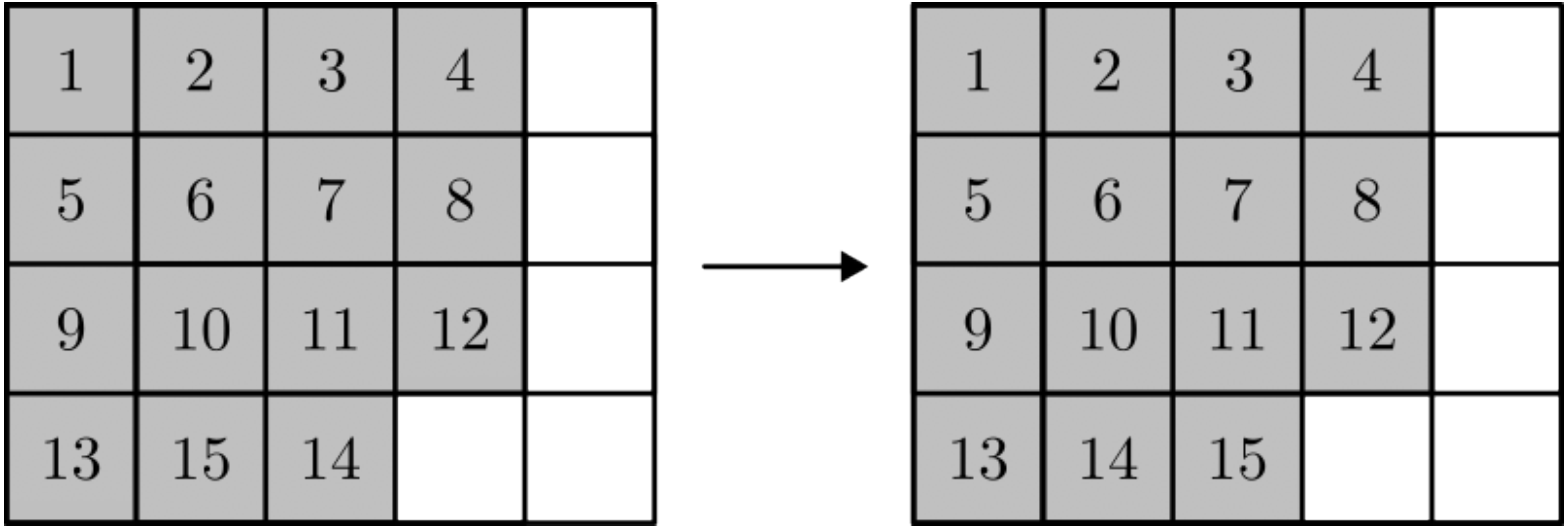

Solving the n Puzzle

Theorem (Johnson–Story 1879)

If $w, h \geq 2$ and $wh - n \geq 2$, then

$$H_0\big(SF_n(R_{w,h})\big) \cong \mathbb{Z}.$$

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What can we say about $H_k\big(SF_n(R_{w,h})\big)$ for $k > 0$?

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If $w, h \geq 2$ and $wh - n \geq 2$, then

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Homological Stability!

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What can we say about $H_k\big(SF_n(R_{w,h})\big)$ for $k > 0$?

Homological Stability

Definition

A sequence $X_1 \to X_2 \to X_3 \to \cdots$ of groups, spaces, etc. exhibits *homological stability* if $H_k(X_n) \to H_k(X_{n+1})$ is an isomorphism for all k and $n \gg k$.

- $\{S_n\}$ Nakaoka 1960
- $\{GL_n(\mathbb{Z})\}$ Borel 1974
- $\{C_n(X)\}$ McDuff 1975, Segal 1979
- $\{\mathrm{Mod}_g\}$ Harer 1985

Homological Stability in the Space Direction

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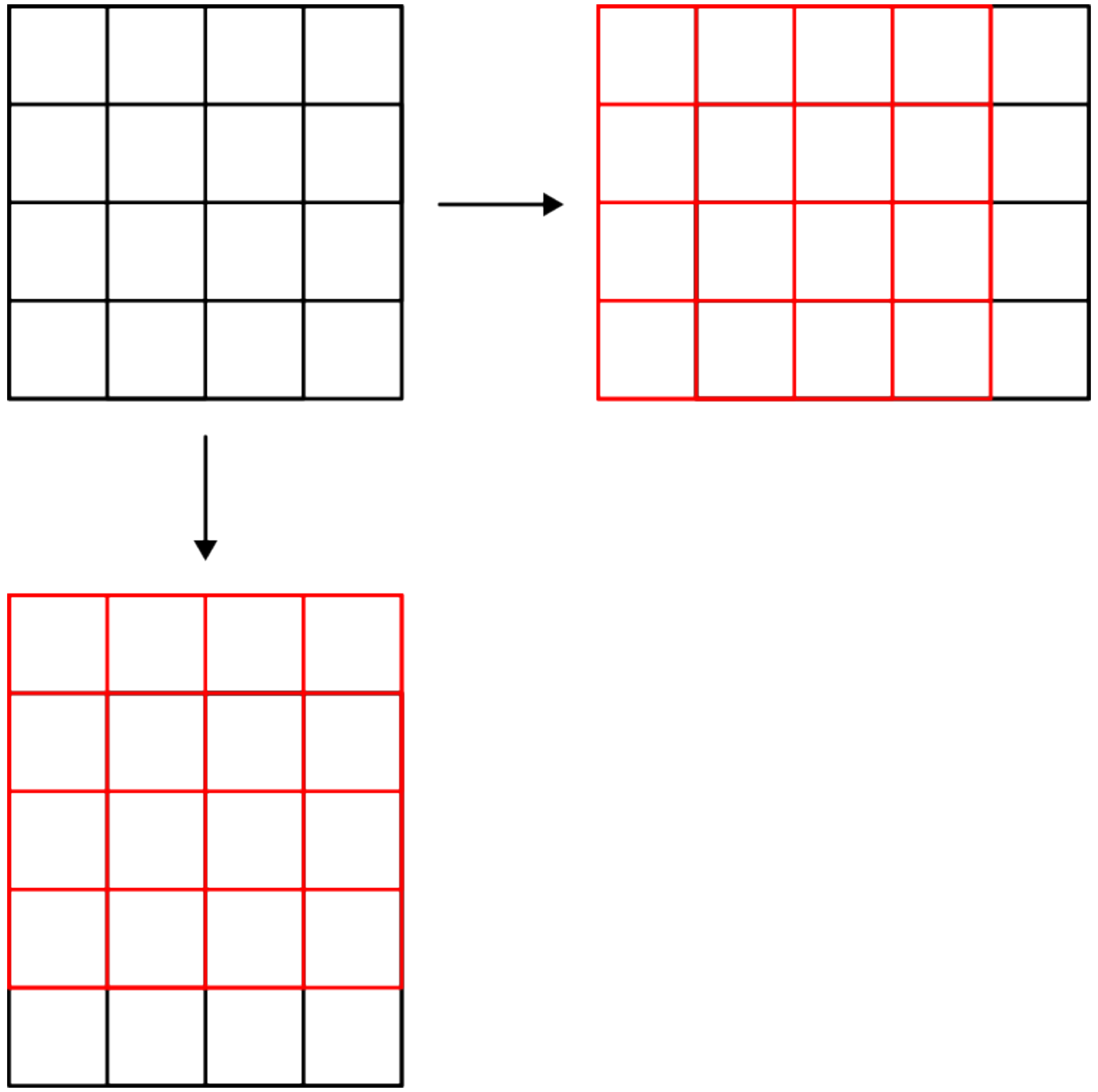
$$\begin{array}{ccc} R_{w,h} & \hookrightarrow & R_{w+1,h} \\ \downarrow & & \\ R_{w,h+1} & & \end{array}$$



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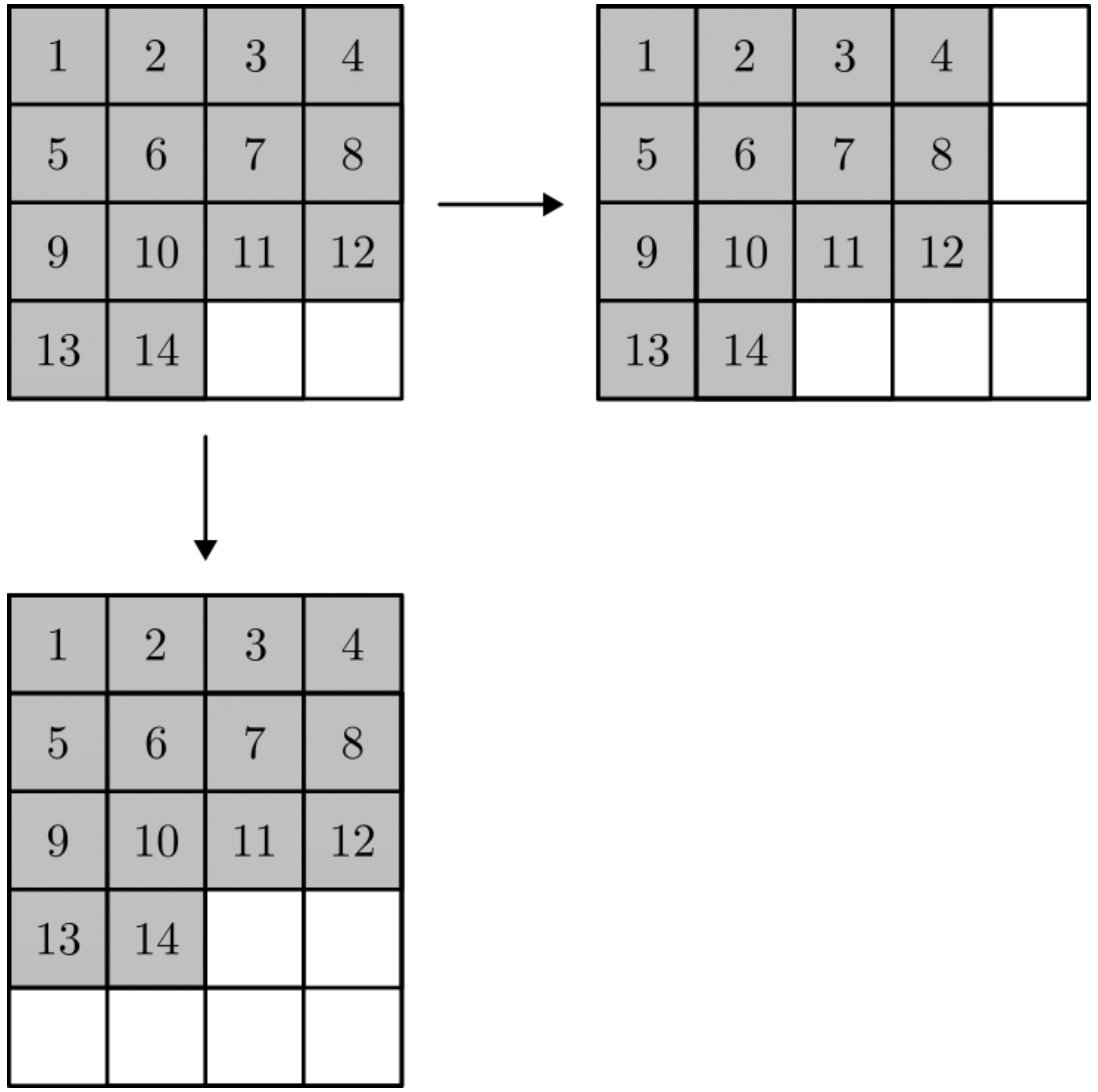

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Question

When, if ever, are these maps on homology isomorphisms?

Homological Stability in the Space Direction

Theorem (Plachta 2021)

$SF_n(R_{w,h}) \simeq F_n(\mathbb{C})$ if and only if $w, h \geq n$.

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$SF_n(R_{w,h}) \simeq F_n(\mathbb{C})$ if and only if $w, h \geq n$.

Corollary

If $w, h \geq n$, the maps on homology are isomorphisms, i.e., $\{SF_n(R_{w,h})\}_{w,h\in\mathbb{N}}$ exhibits homological stability in the space direction.

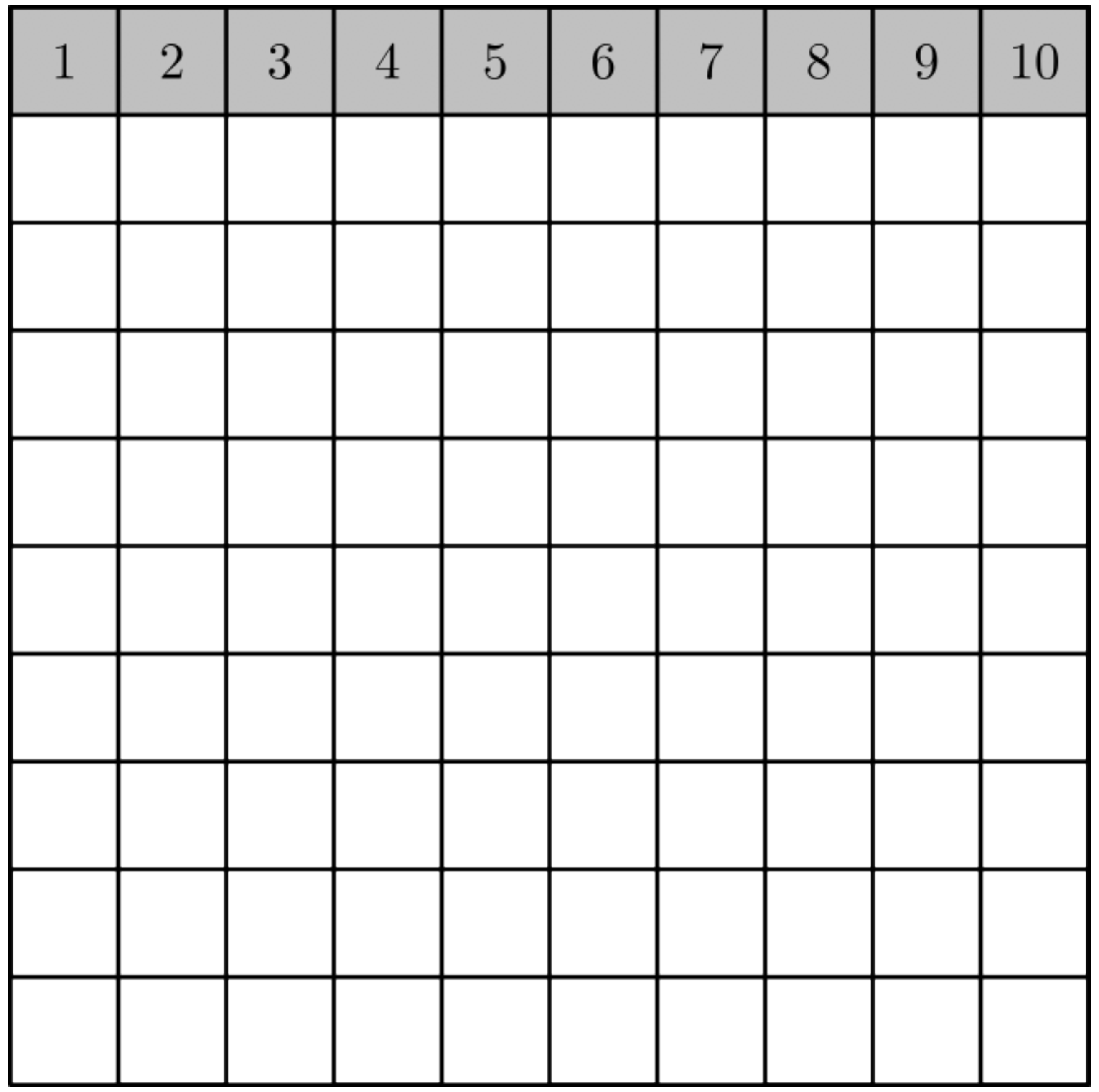
Determining the Stable Range

Question

What is the *stable range*? Alternatively, find conditions on w, h, n, and k that ensure that

$$H_k\big(SF_n(R_{w,h})\big) \cong H_k\big(F_n(\mathbb{C})\big).$$

$SF_n(R_{n,n})$ has a lot of empty space, i.e., $wh - n \approx wh$

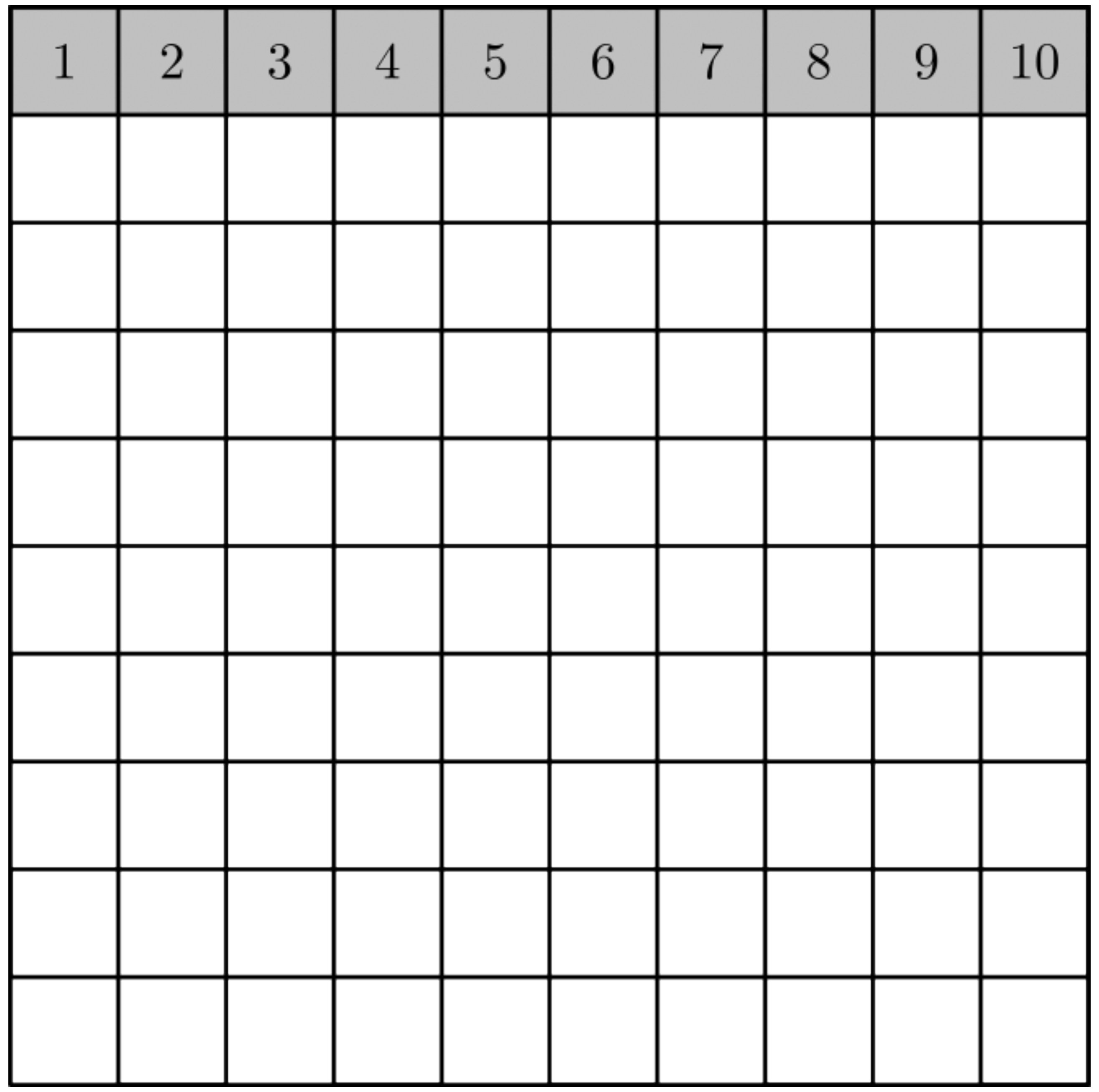


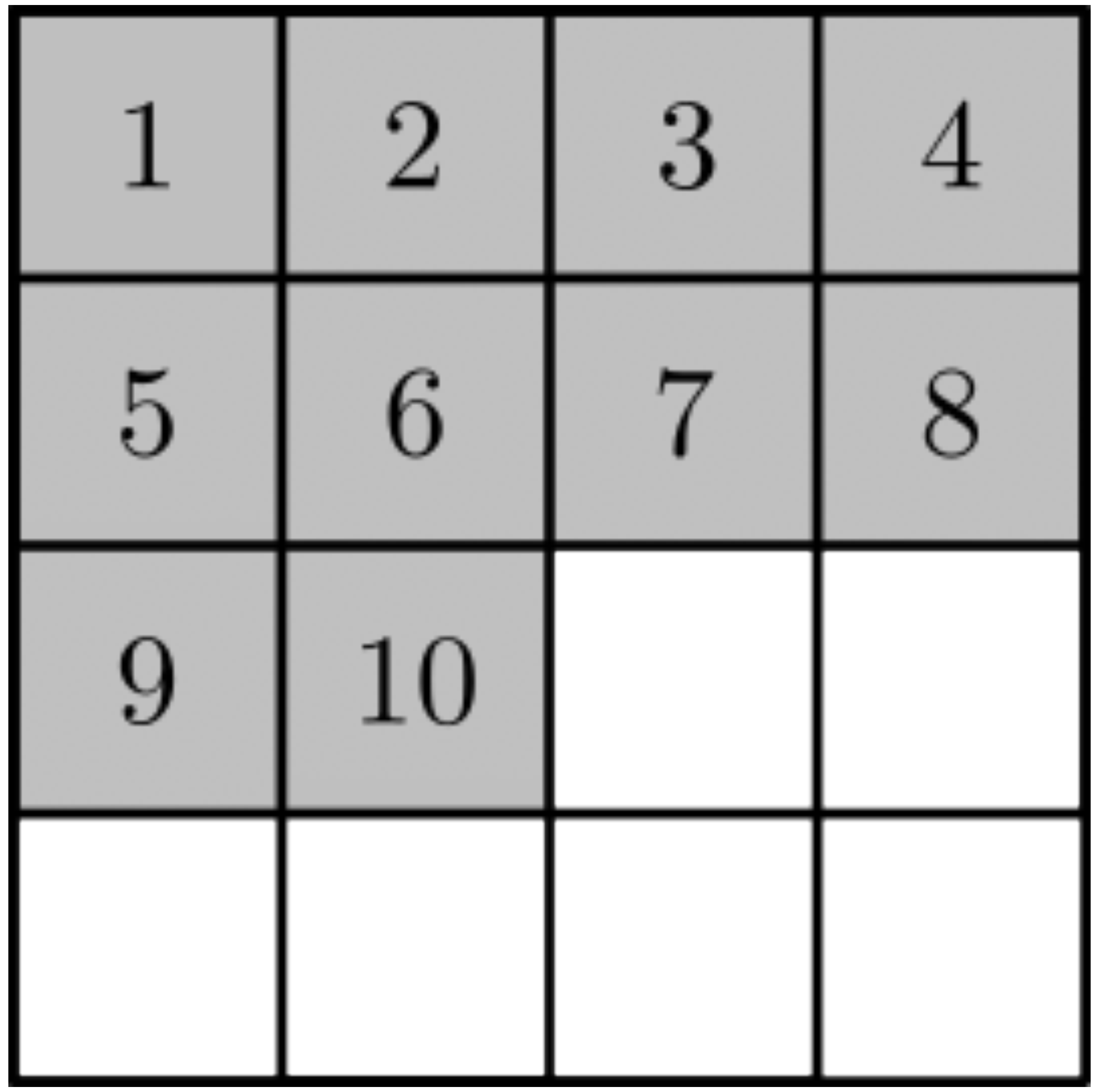
$\mathbf{H_1(SF_n(R_{w,h}))}$

Theorem (González–Kahle–W. 2026)

If $w, h \geq 3$ and $wh - n \geq 6$, then

$$H_1(SF_n(R_{w,h})) \cong H_1(F_n(\mathbb{C})).$$





$\mathbf{H_k\big(SF_n(R_{w,h})\big)}$

Theorem (González–Kahle–W. 2026)

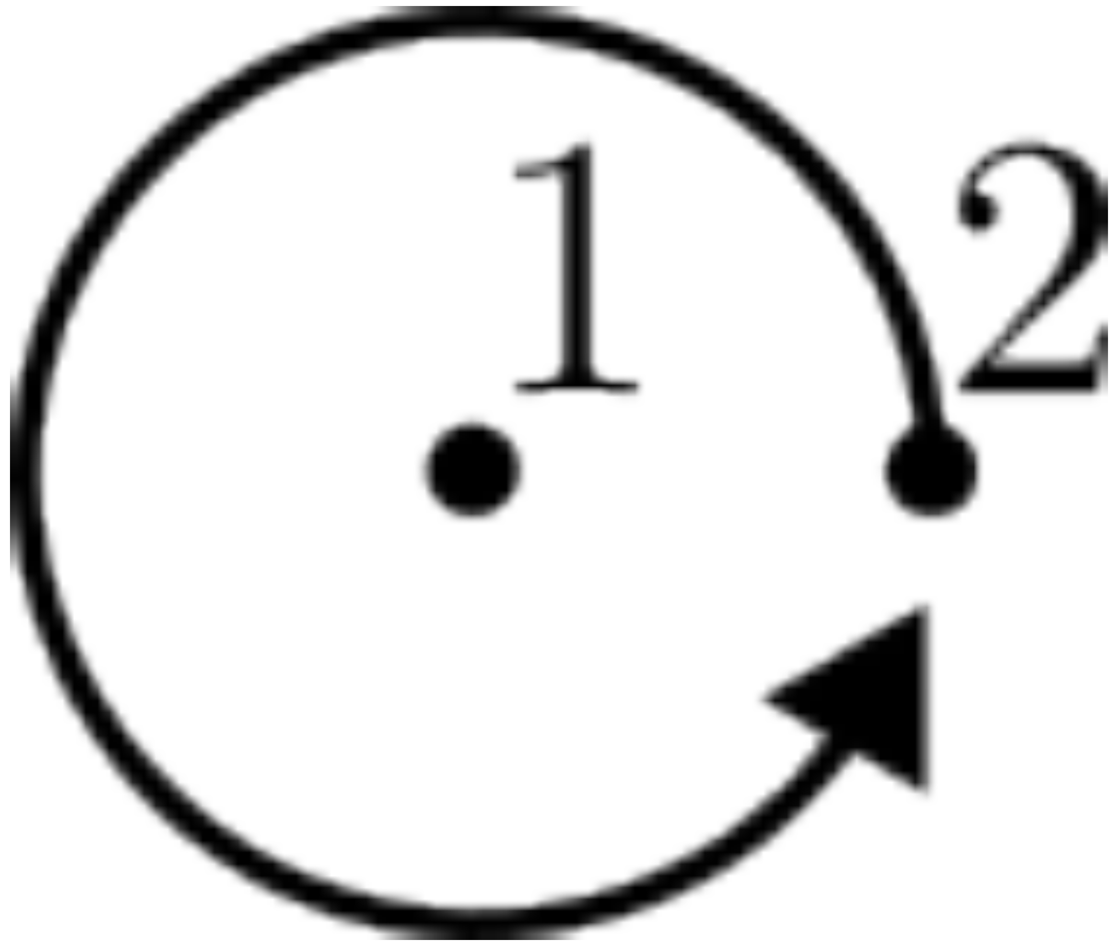
If $k \geq 2$, $w \geq h \geq k+2$, and $wh - n \geq \max\{(k+1)(k+2), hk+2\}$, then

$$H_k\big(SF_n(R_{w,h})\big) \cong H_k\big(F_n(\mathbb{C})\big).$$

$\implies$ For big w and h, almost all of $R_{w,h}$ can be filled with squares and there is still an isomorphism $H_k\big(SF_n(R_{w,h})\big) \cong H_k\big(F_n(\mathbb{C})\big)$.

Why Expect this Range?

$$F_2(\mathbb{C}) \simeq S^1 \implies H_1\big(F_2(\mathbb{C})\big) \cong \mathbb{Z}$$



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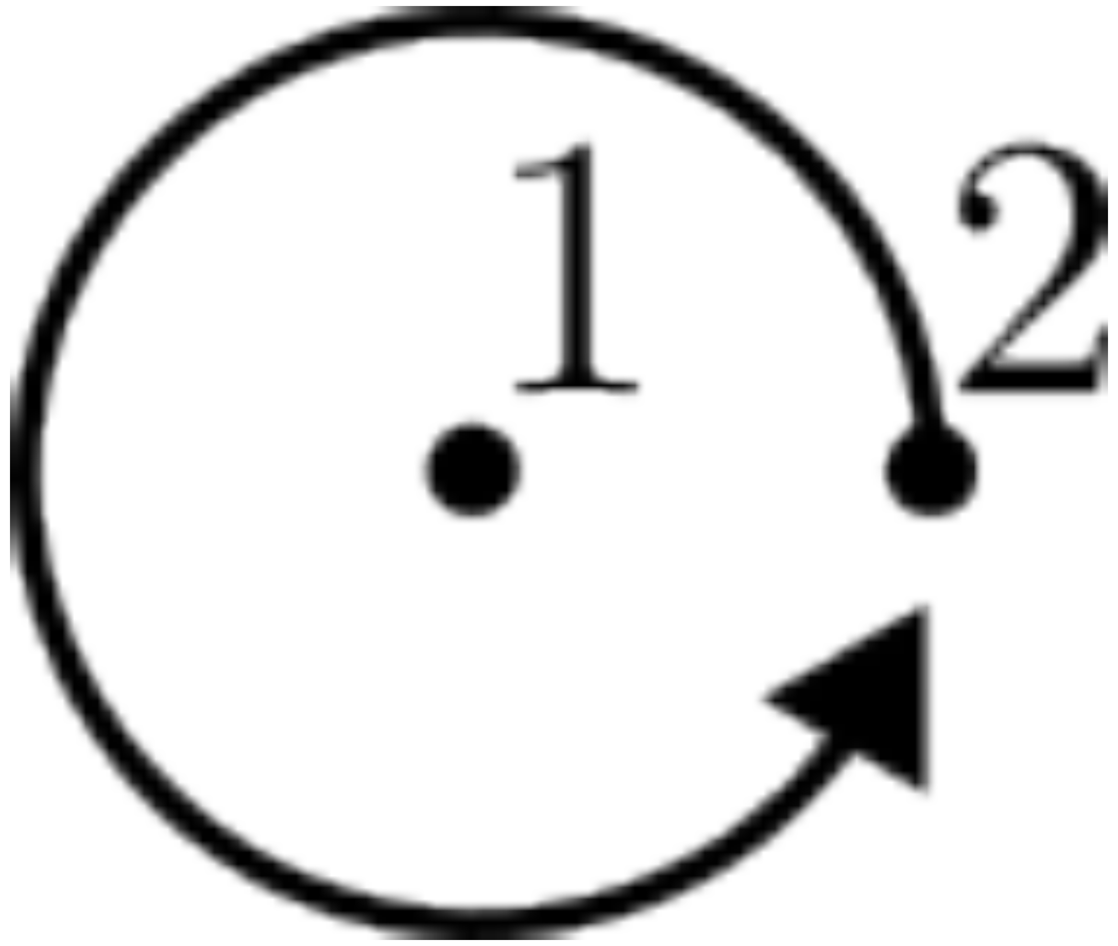
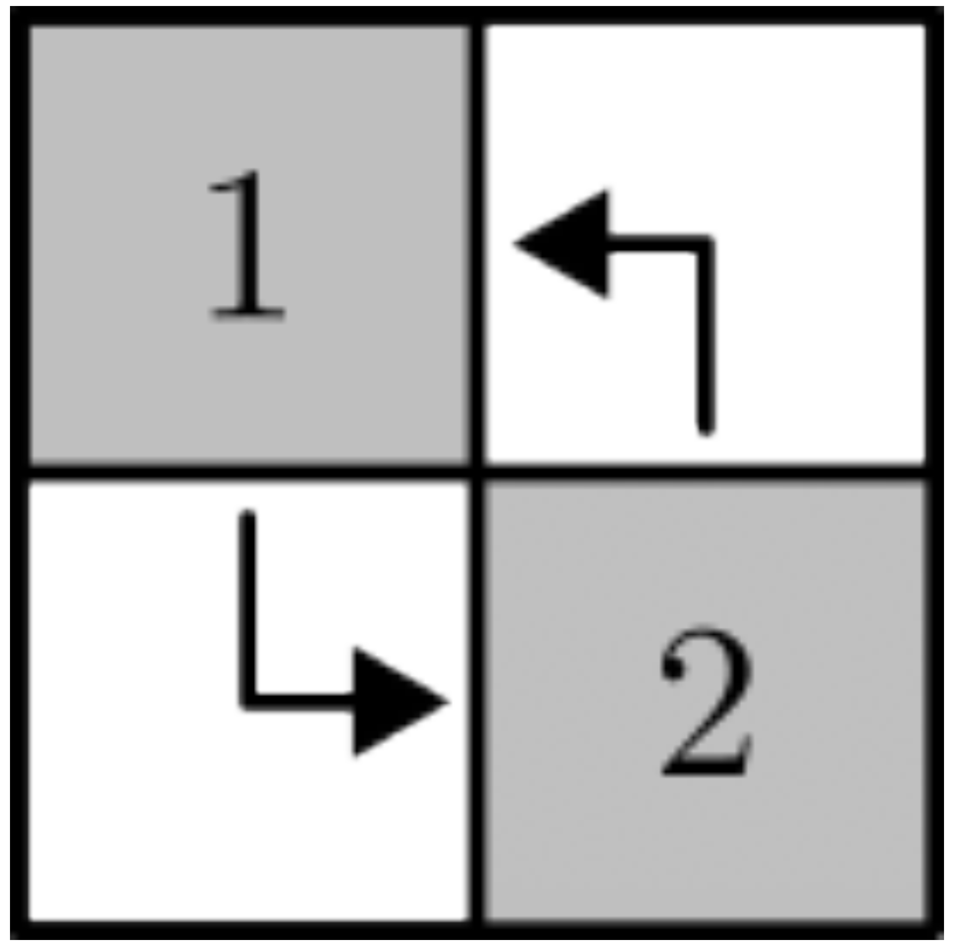
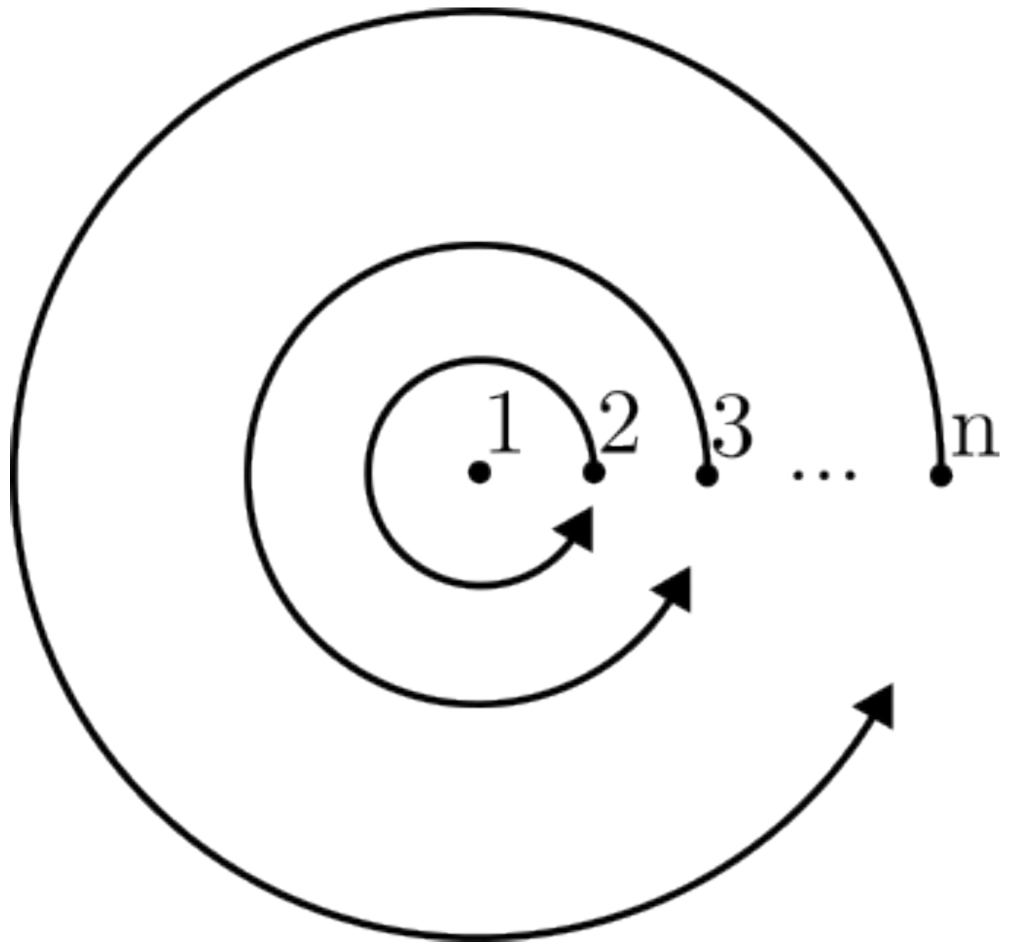


$$SF_2(R_{2,2}) \simeq F_2(\mathbb{C}) \implies H_1\big(SF_2(R_{2,2})\big) \cong \mathbb{Z}$$



Why Expect this Range?

$H_{n-1}\big(F_n(\mathbb{C})\big)$ is generated by



$SF_n(R_{n,n}) \simeq F_n(\mathbb{C}) \implies$
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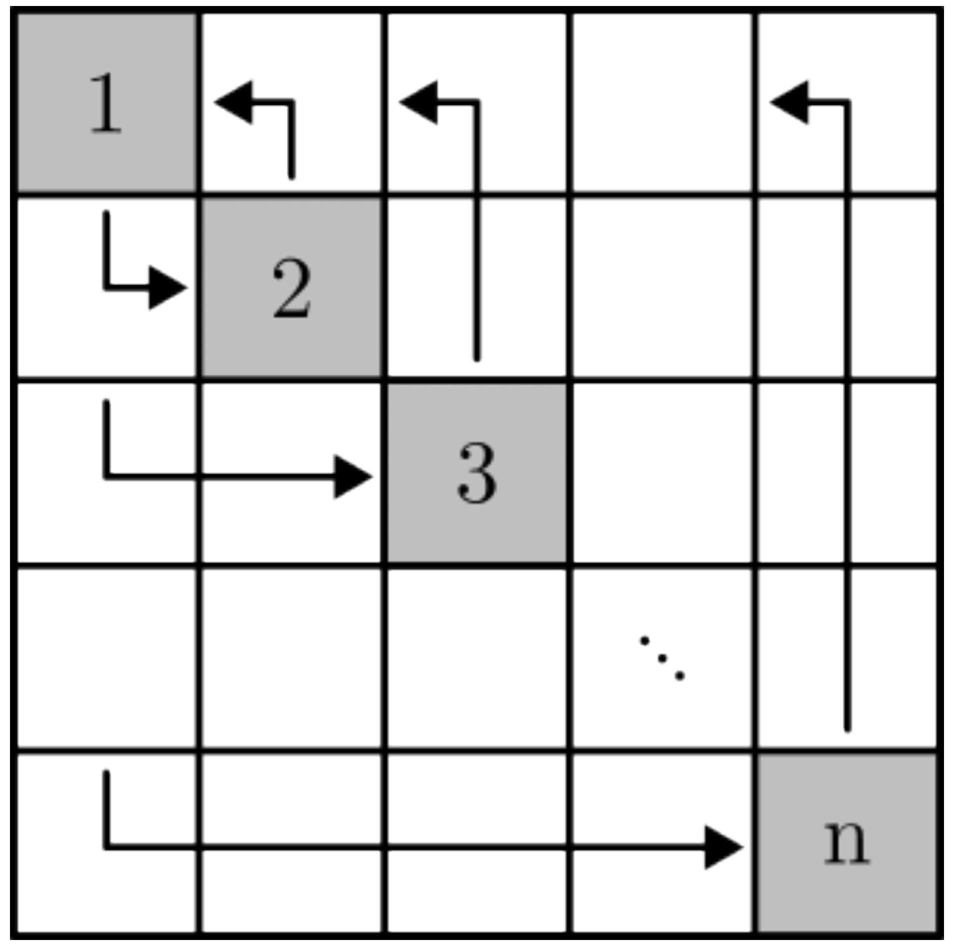
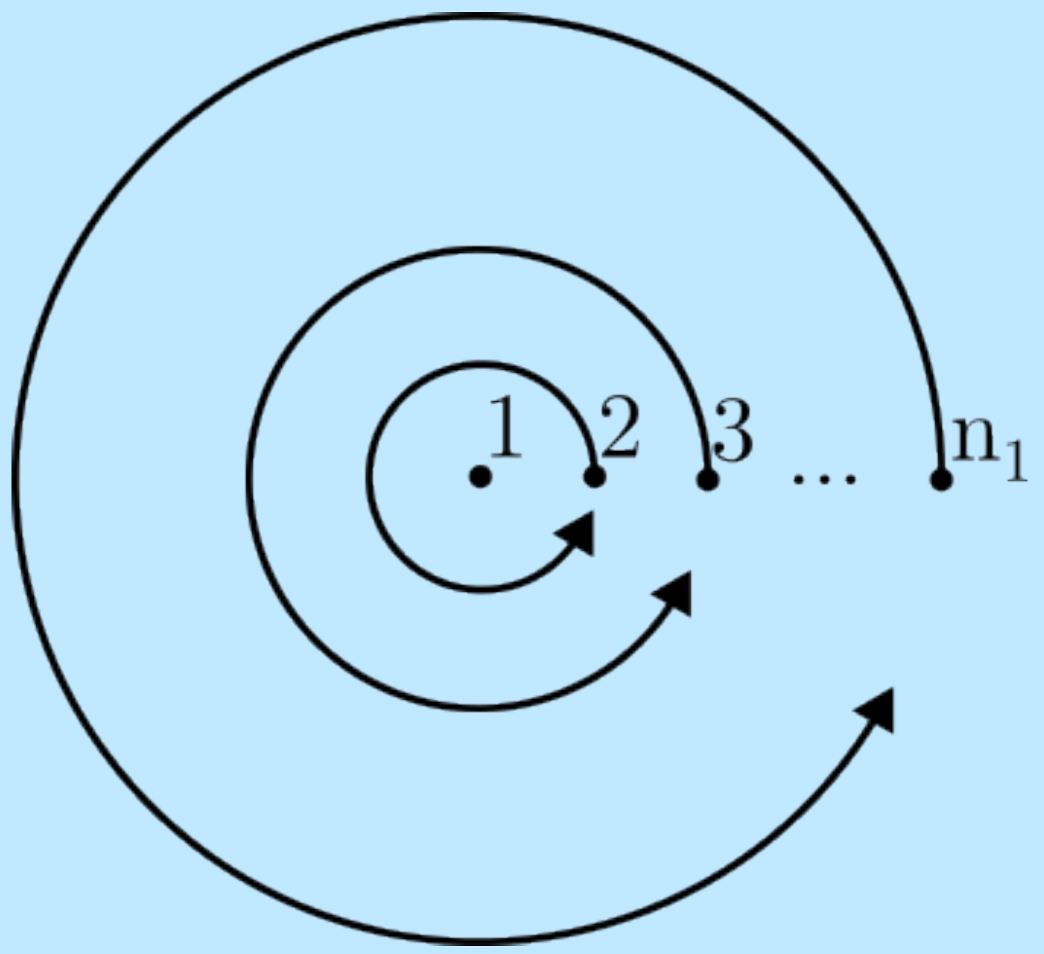
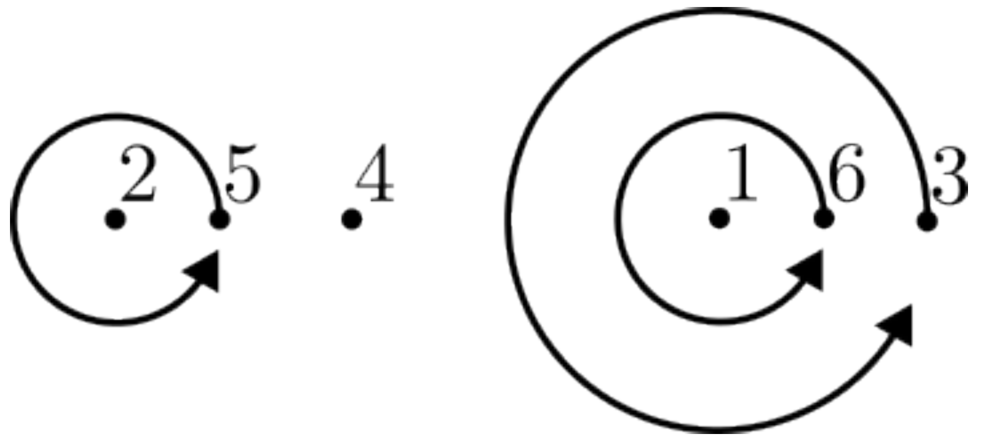


Generating $\mathbf{H_k(F_n(\mathbb{C}))}$

Theorem (Arnol'd 1969, Cohen 1976)

$H_k(F_n(\mathbb{C}))$ is spanned by products of the





$\in H_3(F_6(\mathbb{C}))$

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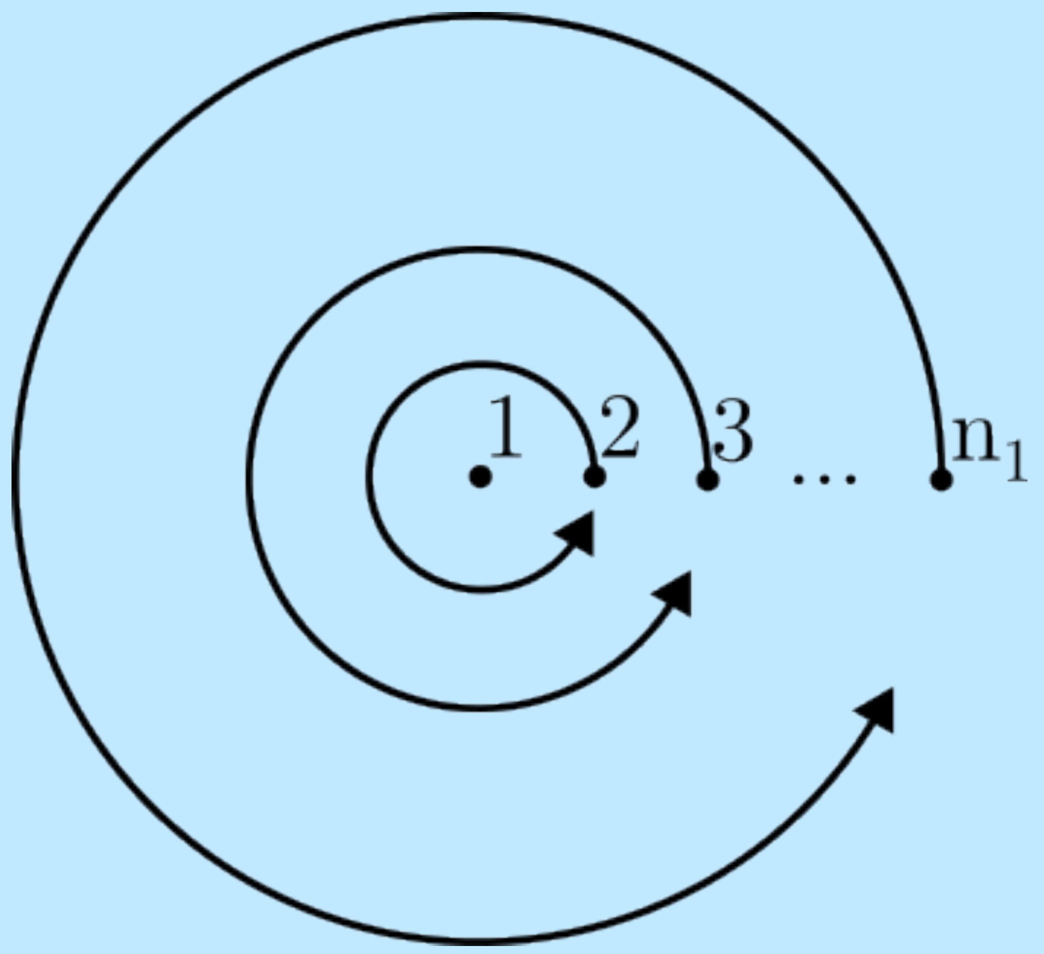
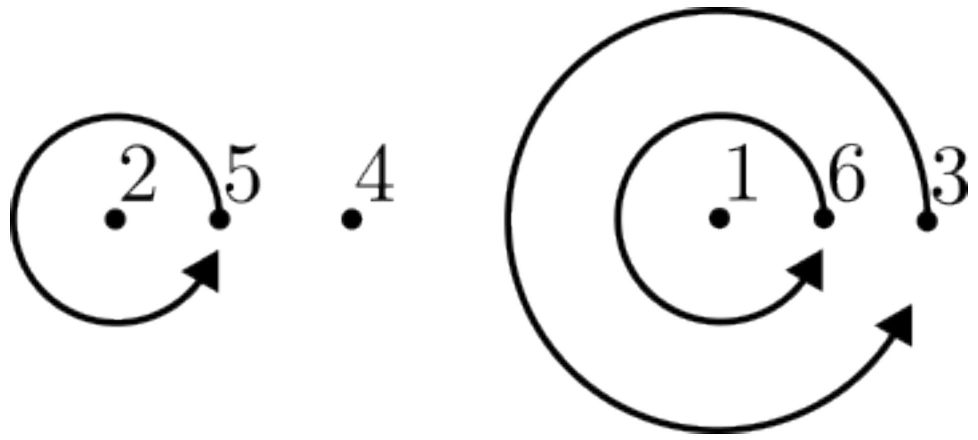


These classes exist in $H_k(SF_n(R_{w,h}))$ if $w, h \geq k+2$ and $wh - n \geq (k+1)(k+2)$



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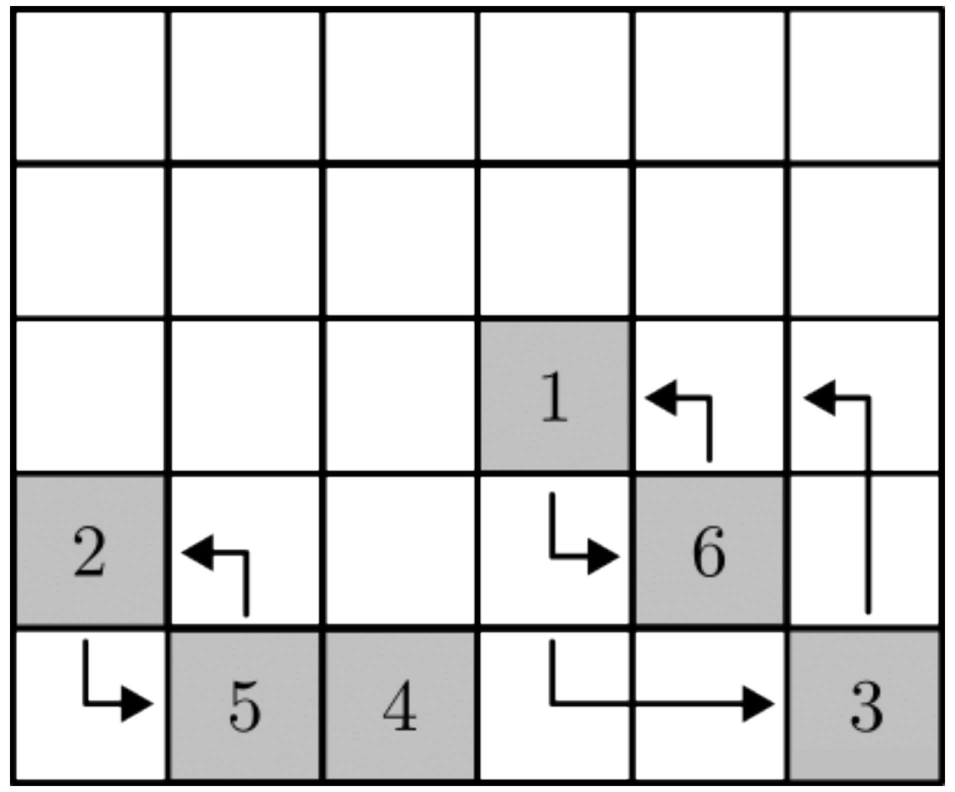
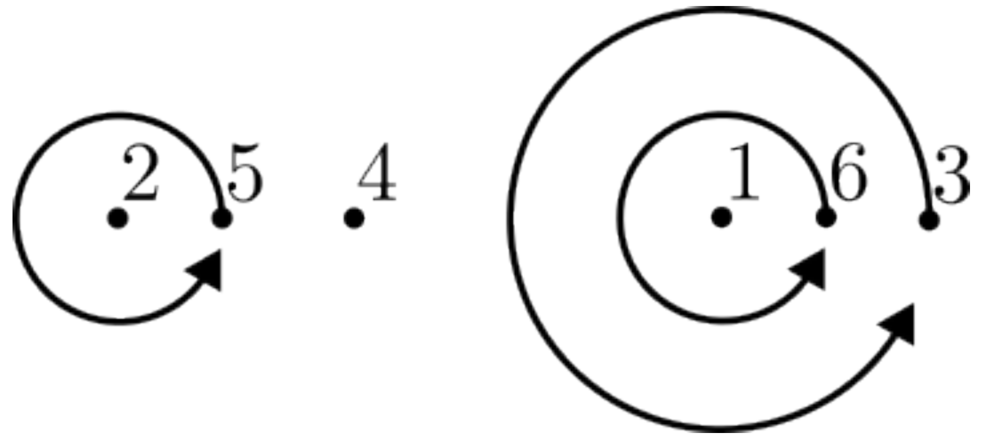
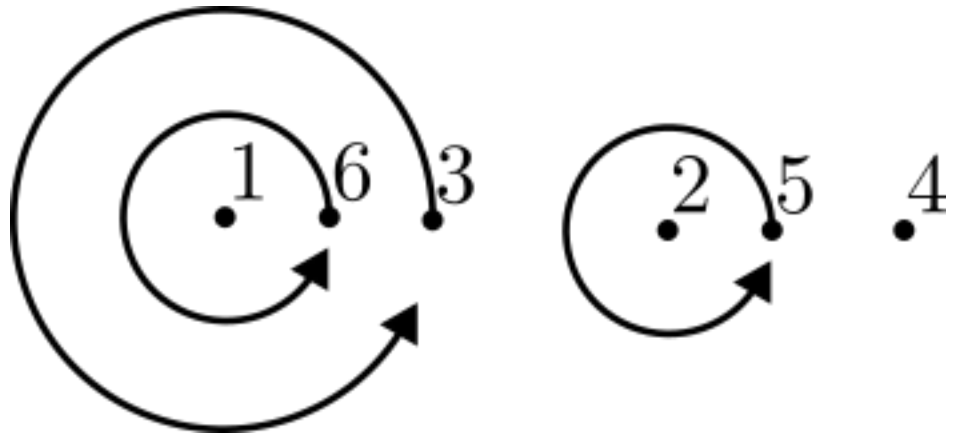


$\in H_3(SF_6(R_{6,5}))$

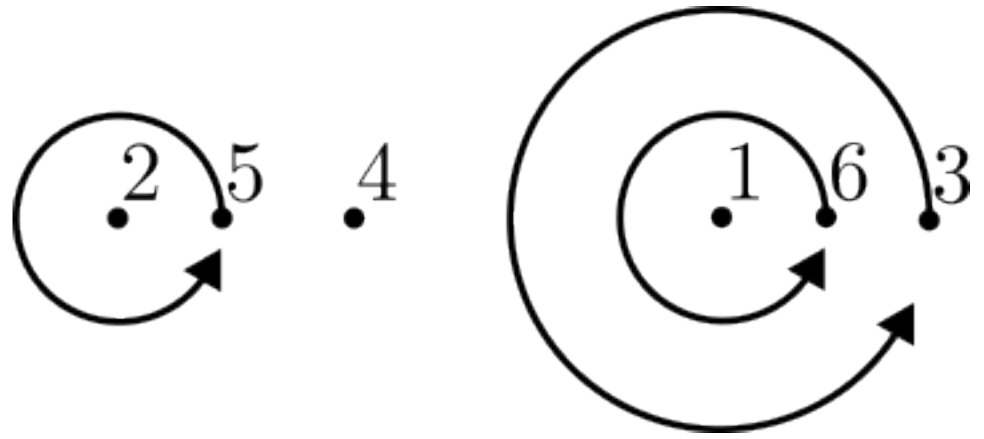
Relations in $\mathbf{H_k\big(F_n(\mathbb{C})\big)}$

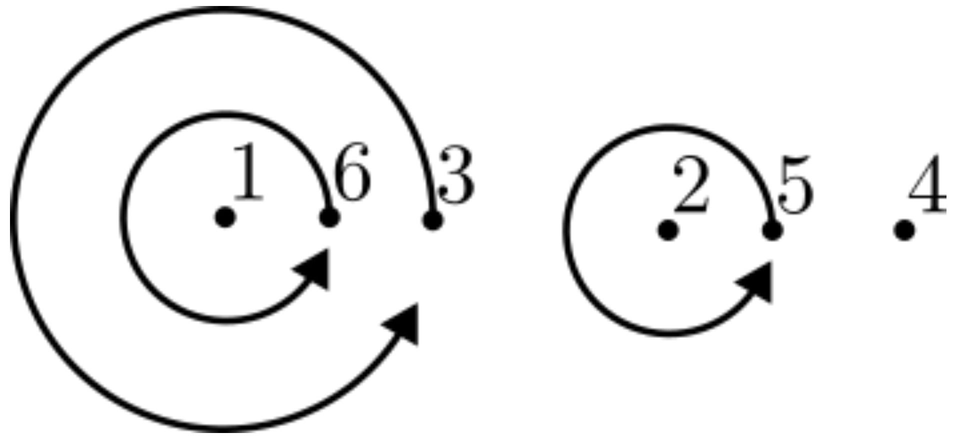




The same class in $H_3\big(F_6(\mathbb{C})\big)$

Relations in $\mathbf{H_k(F_n(\mathbb{C}))}$





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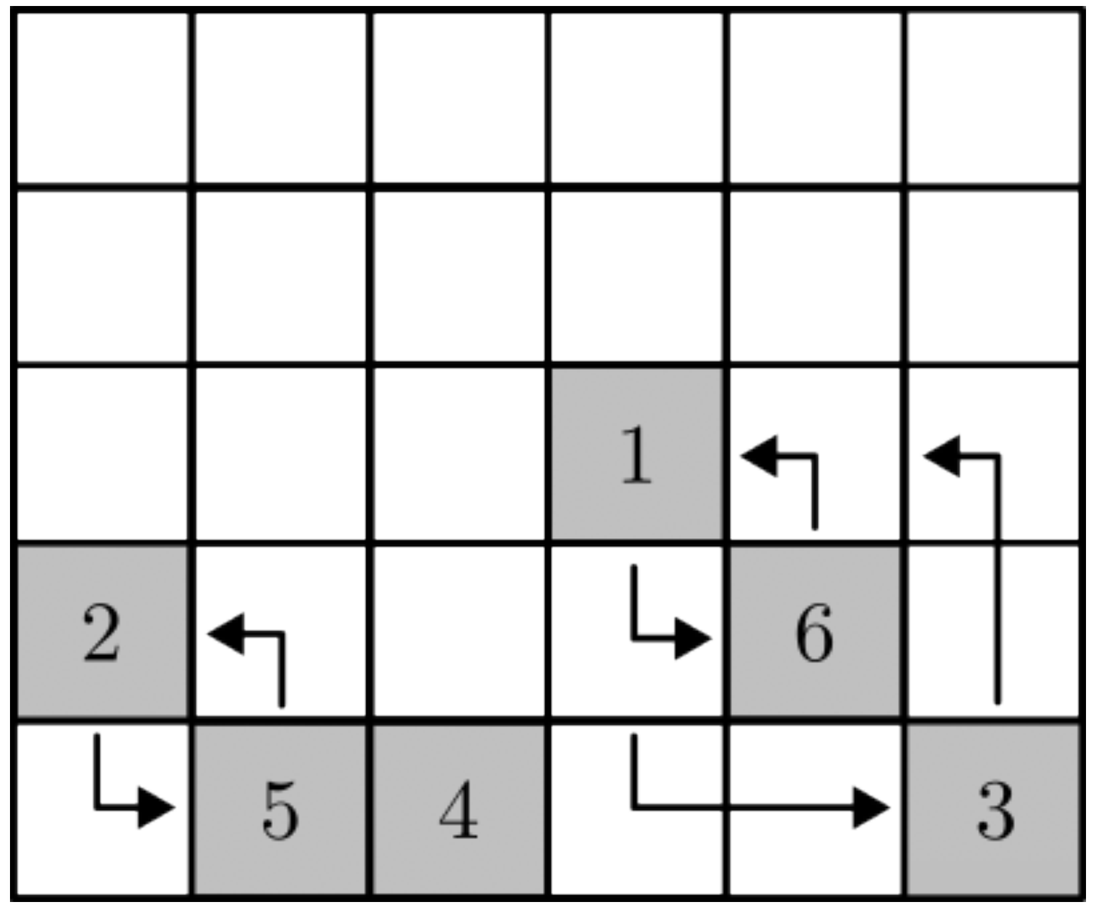
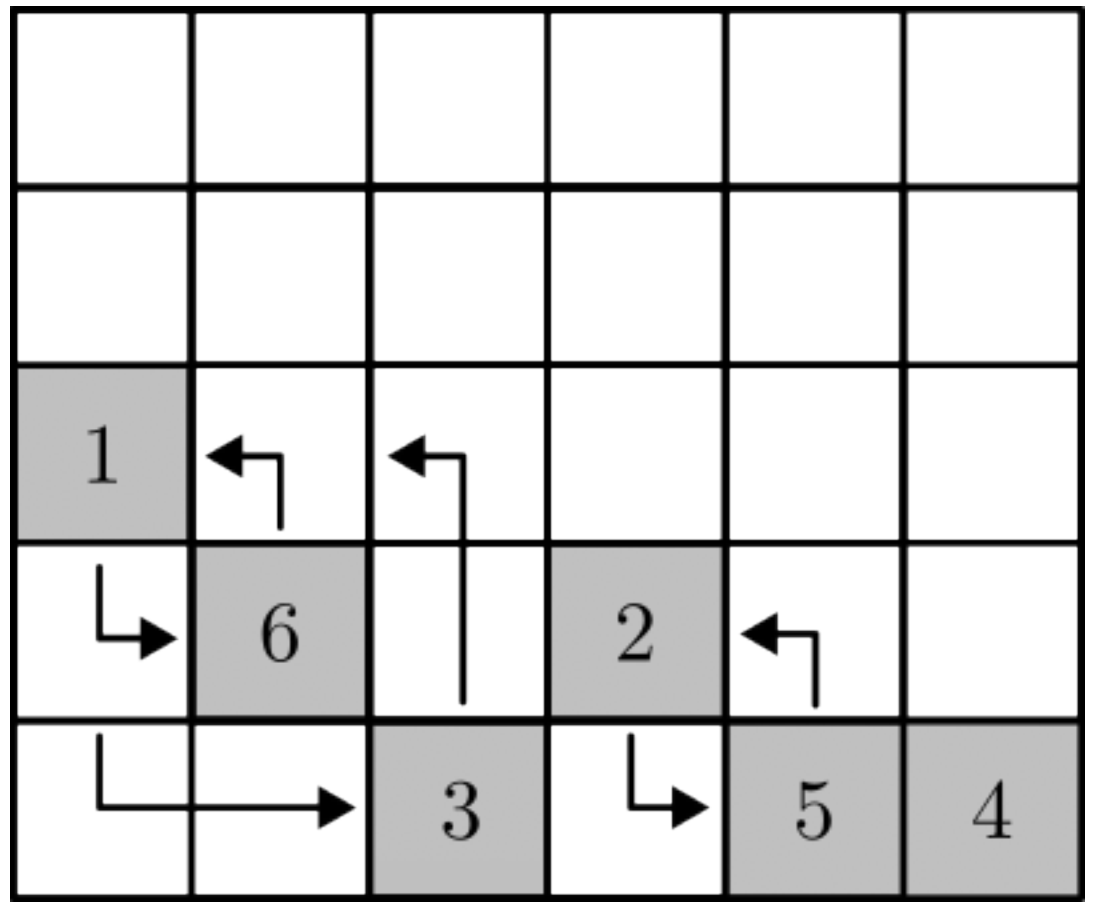
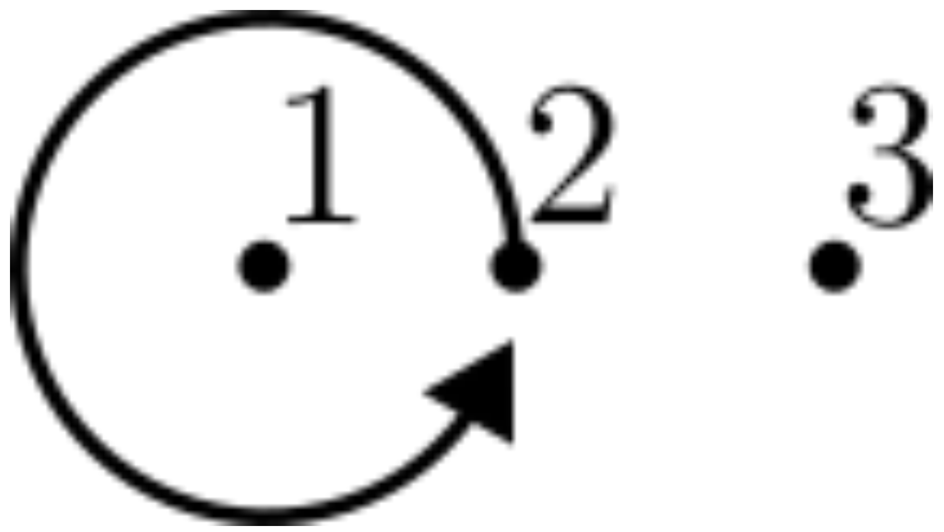




The same class in $H_3(SF_6(R_{6,5}))$

Why this Range is Surprising

$H_1\big(F_3(\mathbb{C})\big) \cong \mathbb{Z}^3$



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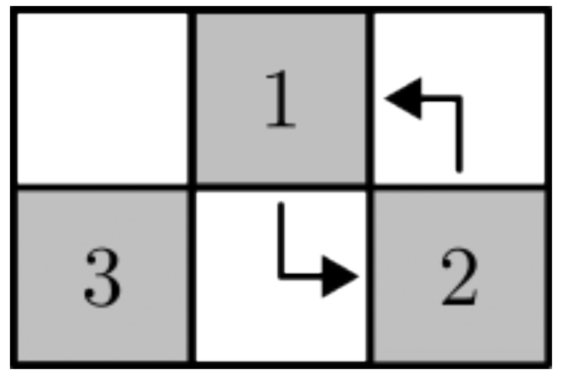

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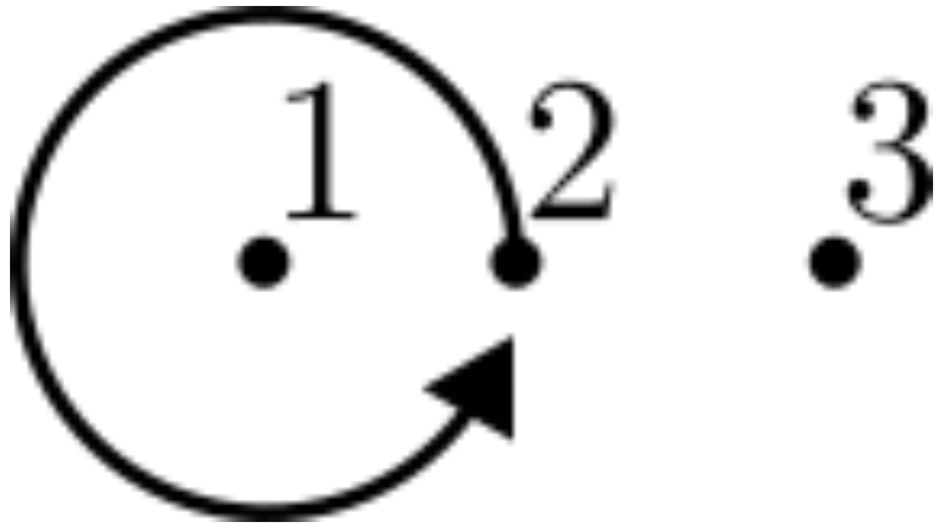

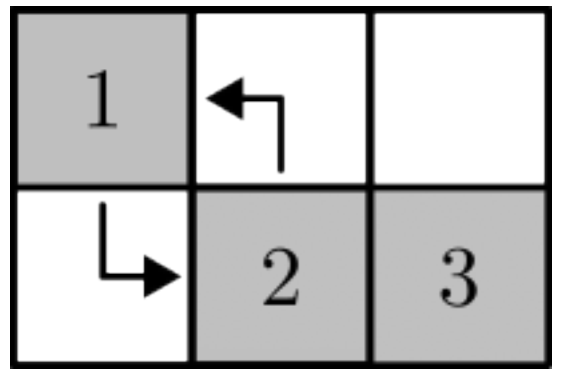

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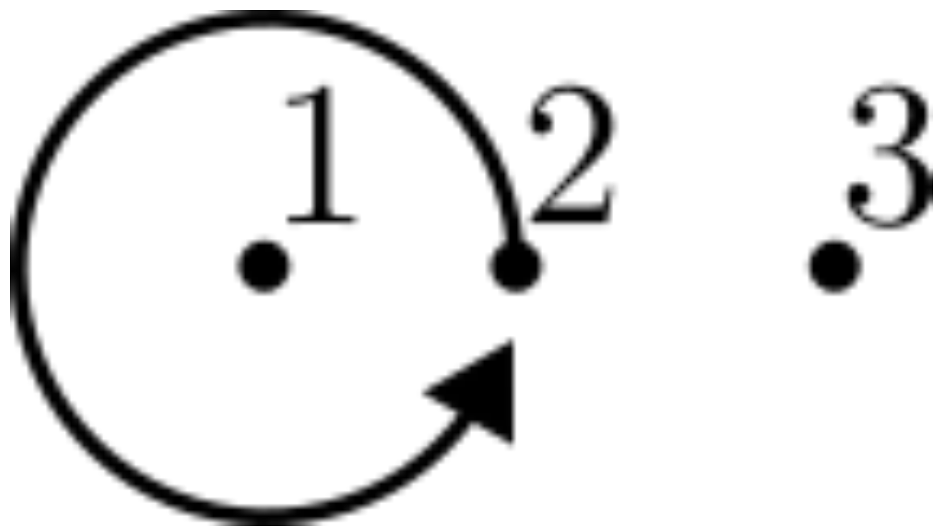

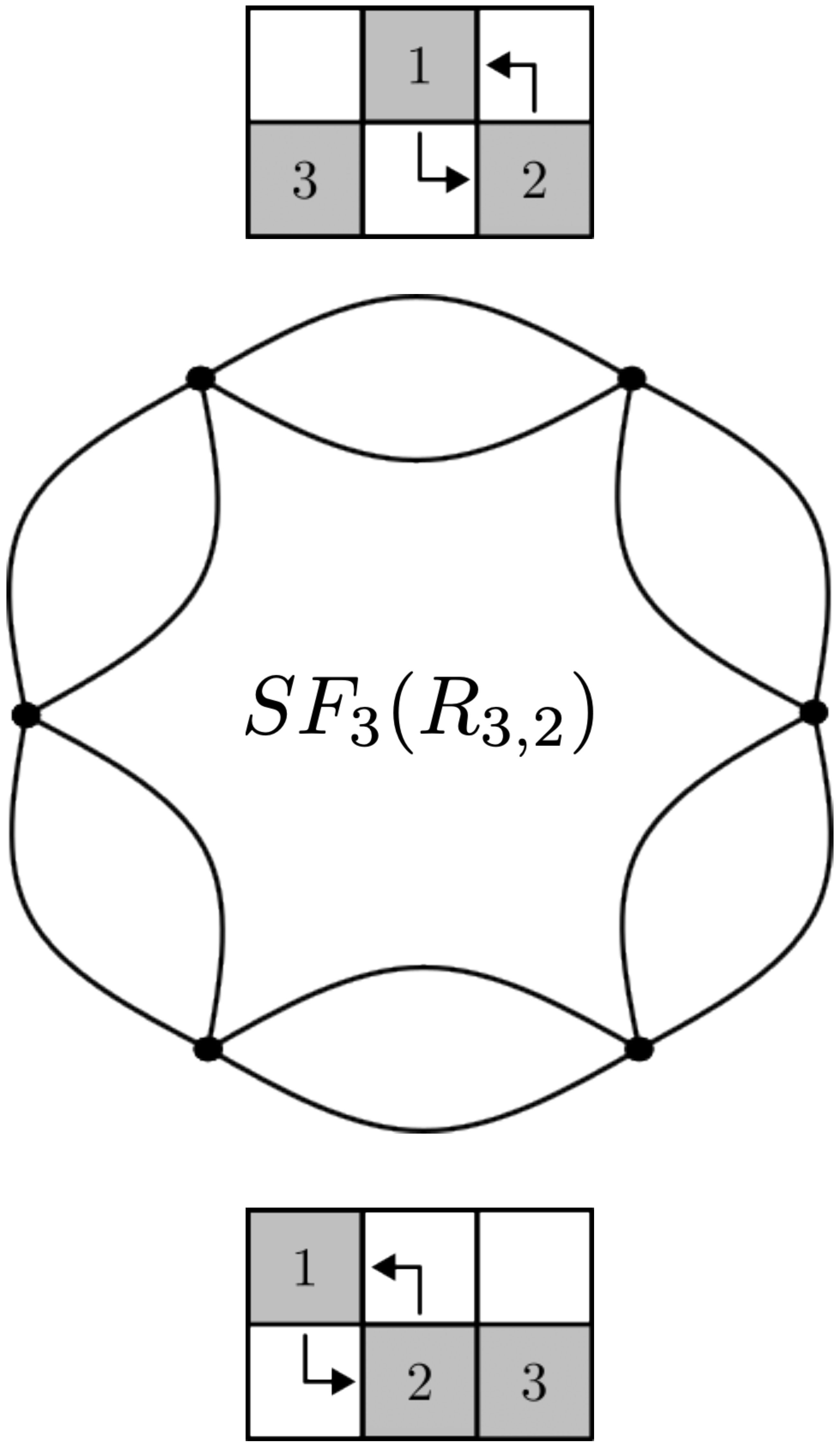

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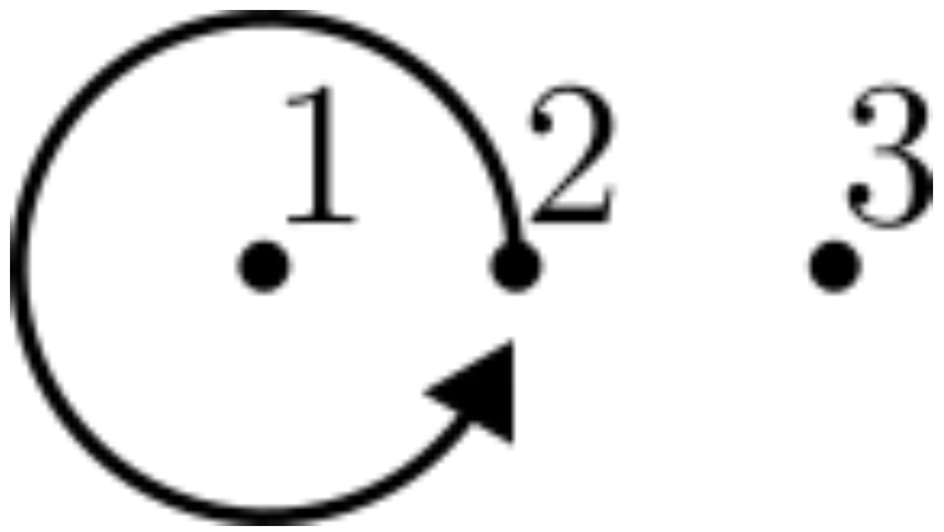

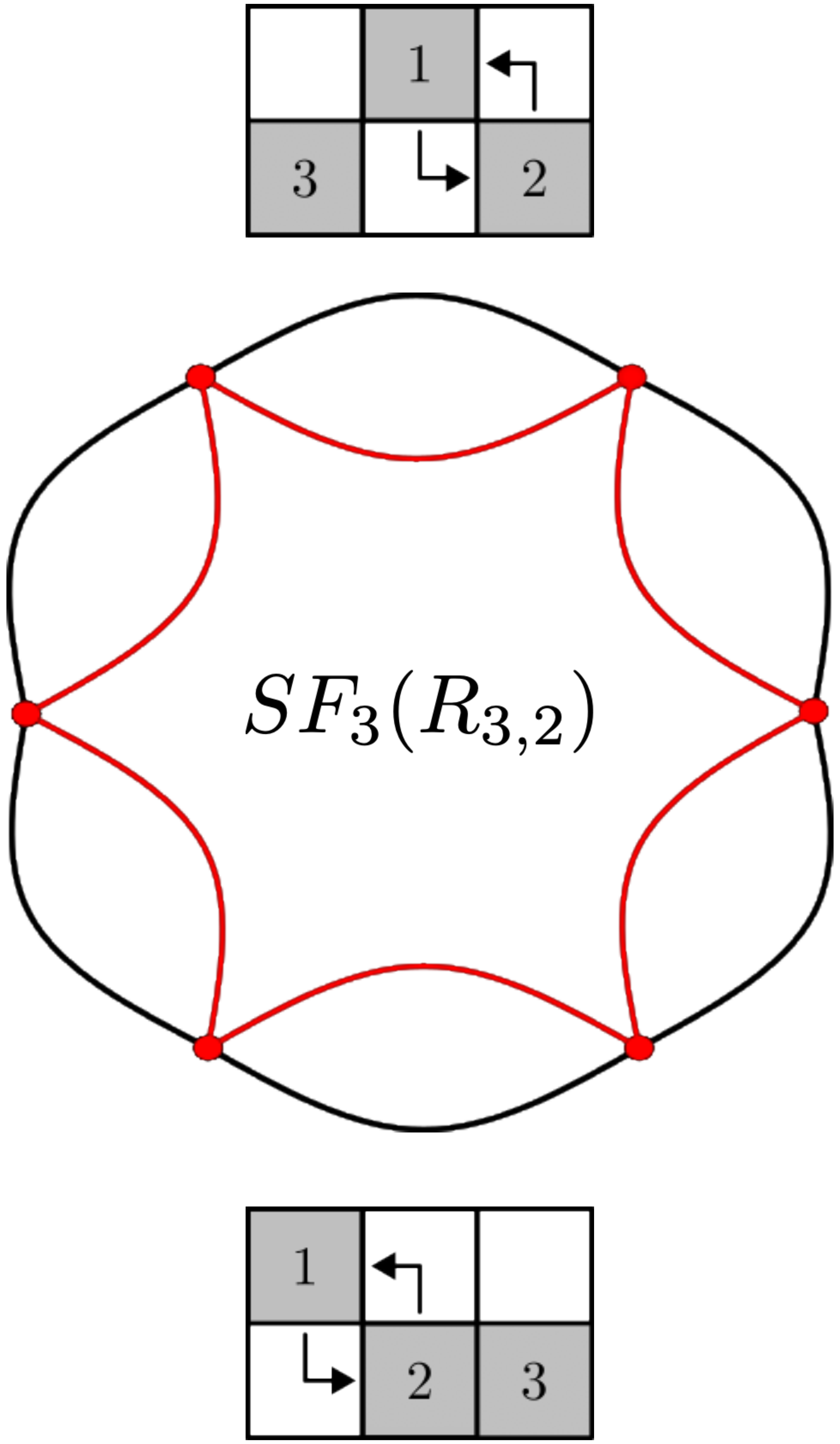

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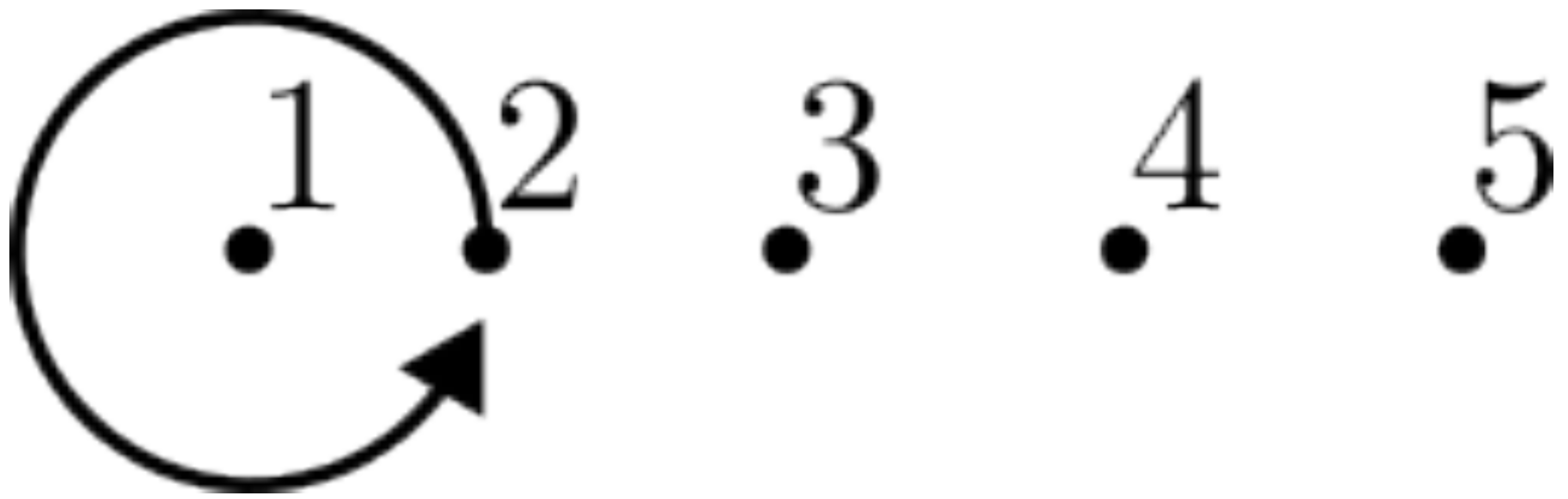
$H_1\big(F_5(\mathbb{C});\mathbb{Q}\big) \cong \mathbb{Q}^{10}$



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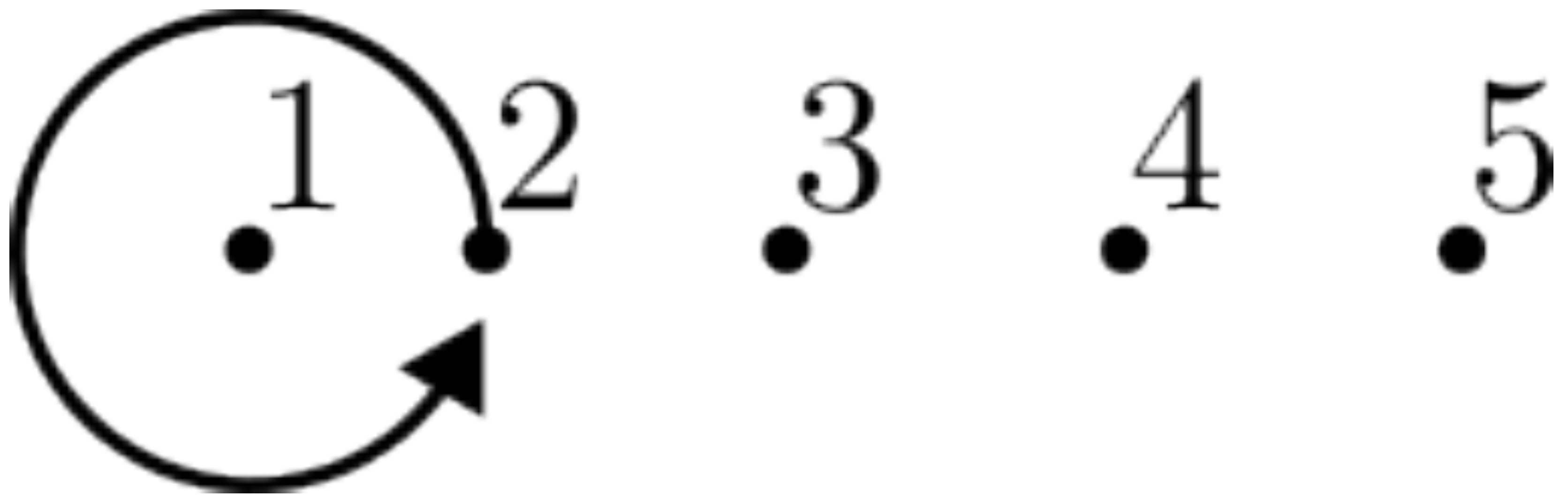
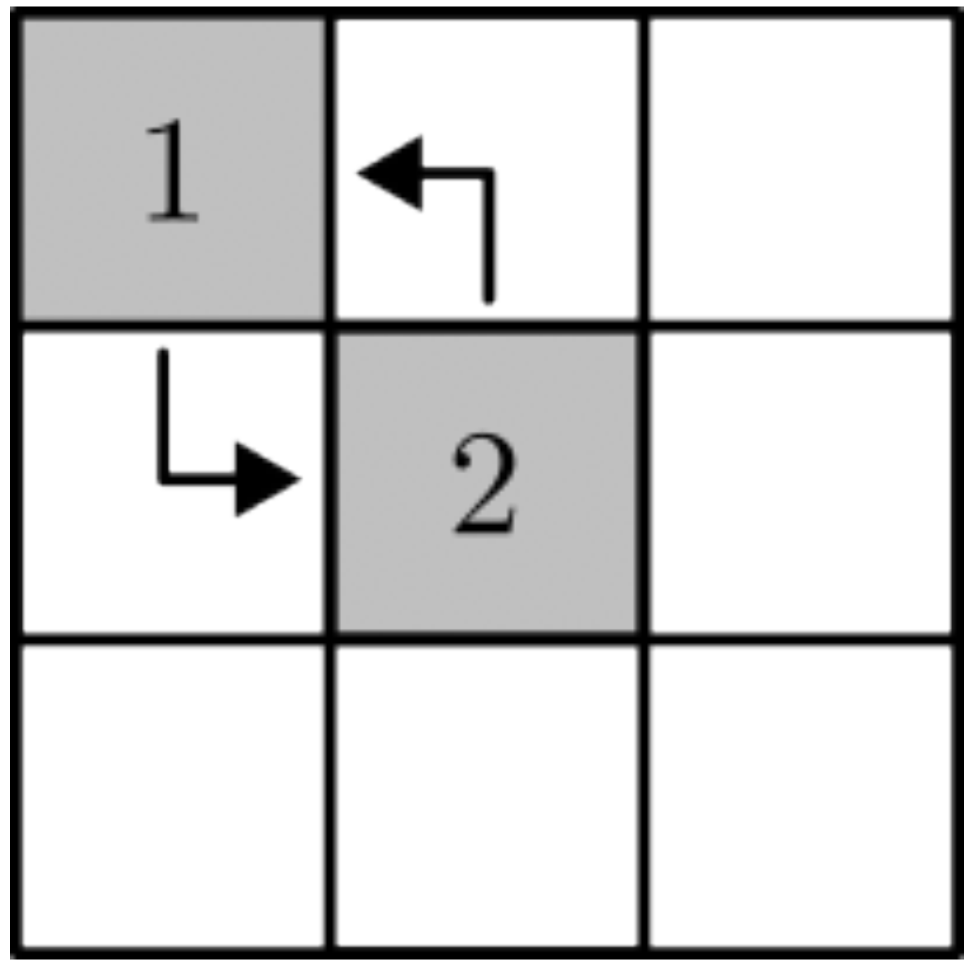
$H_1\big(F_5(\mathbb{C});\mathbb{Q}\big) \cong \mathbb{Q}^{10}$

$H_1\big(SF_5(R_{3,3});\mathbb{Q}\big) \cong \mathbb{Q}^{68} \cong \mathbb{Q}^{60} \oplus \mathbb{Q}^{8}$





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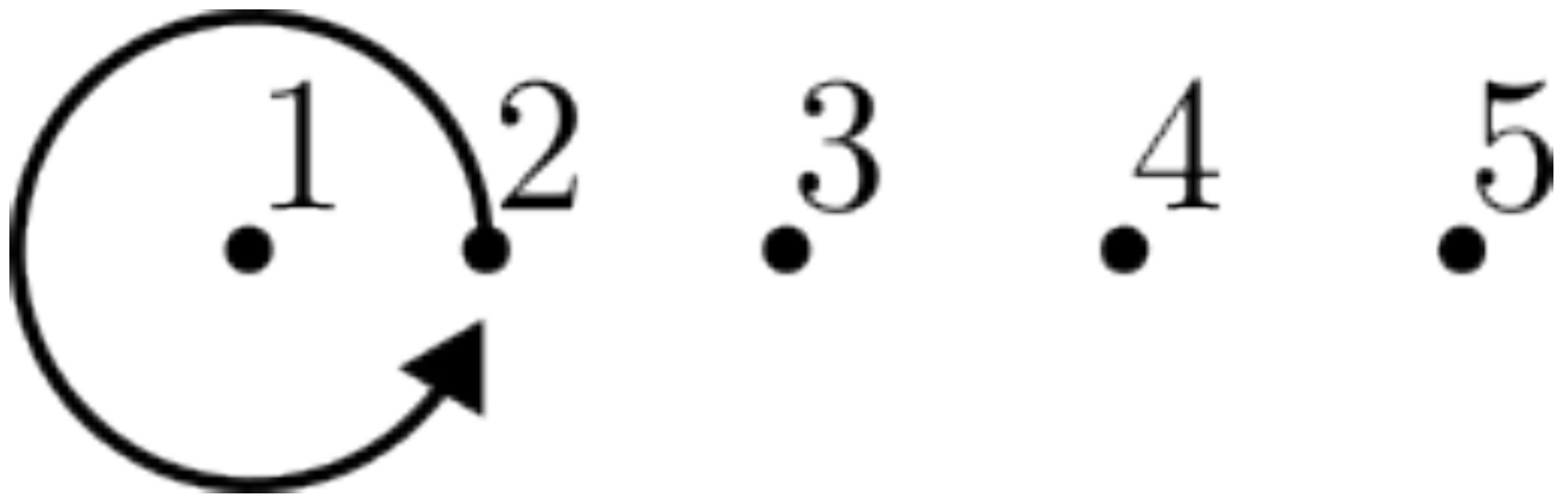

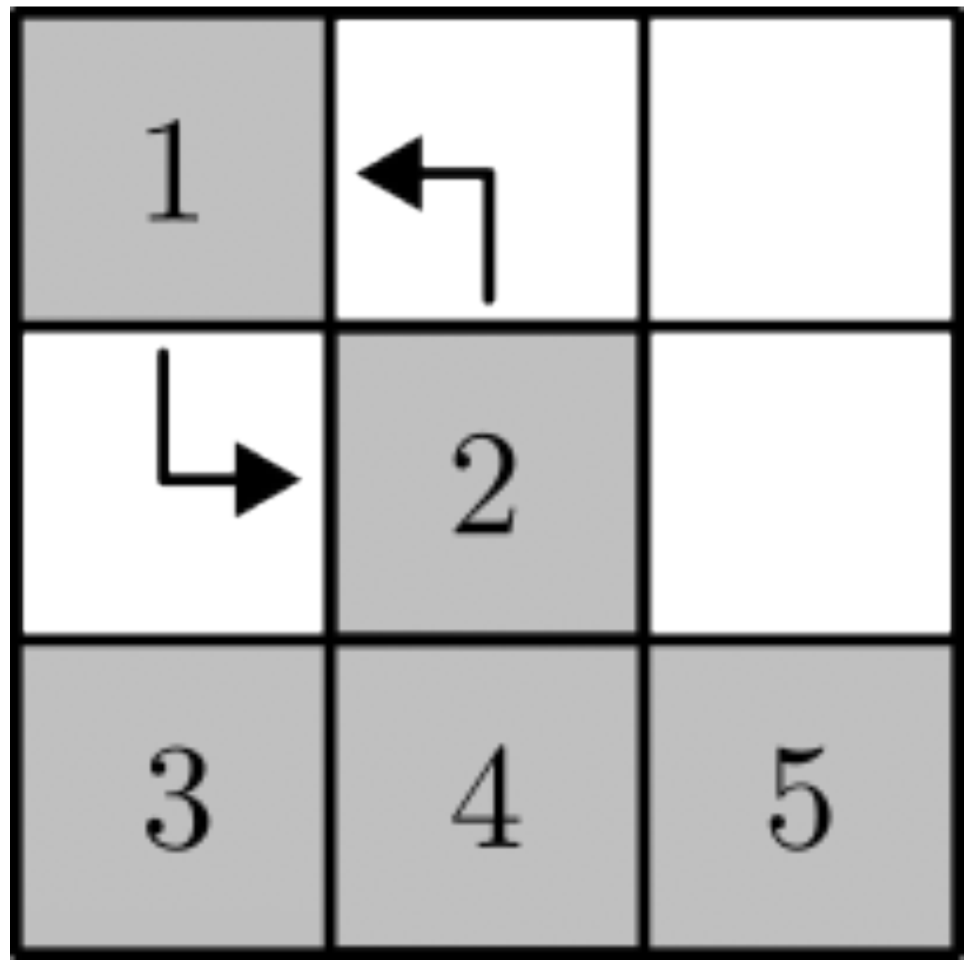

Why this Range is Surprising

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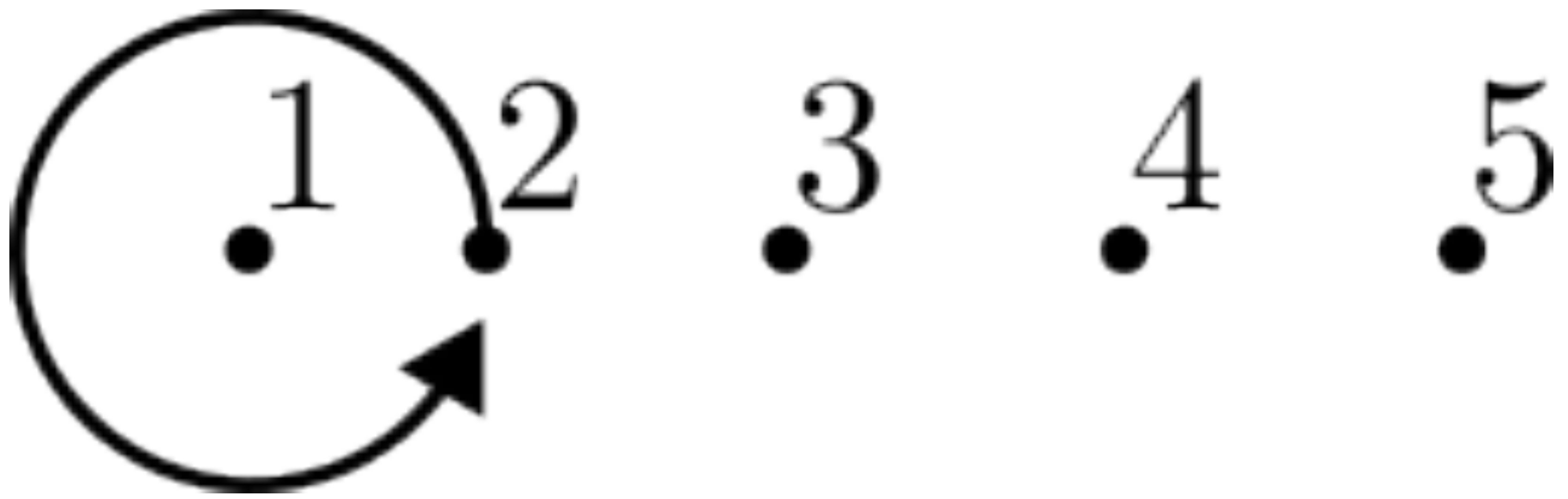

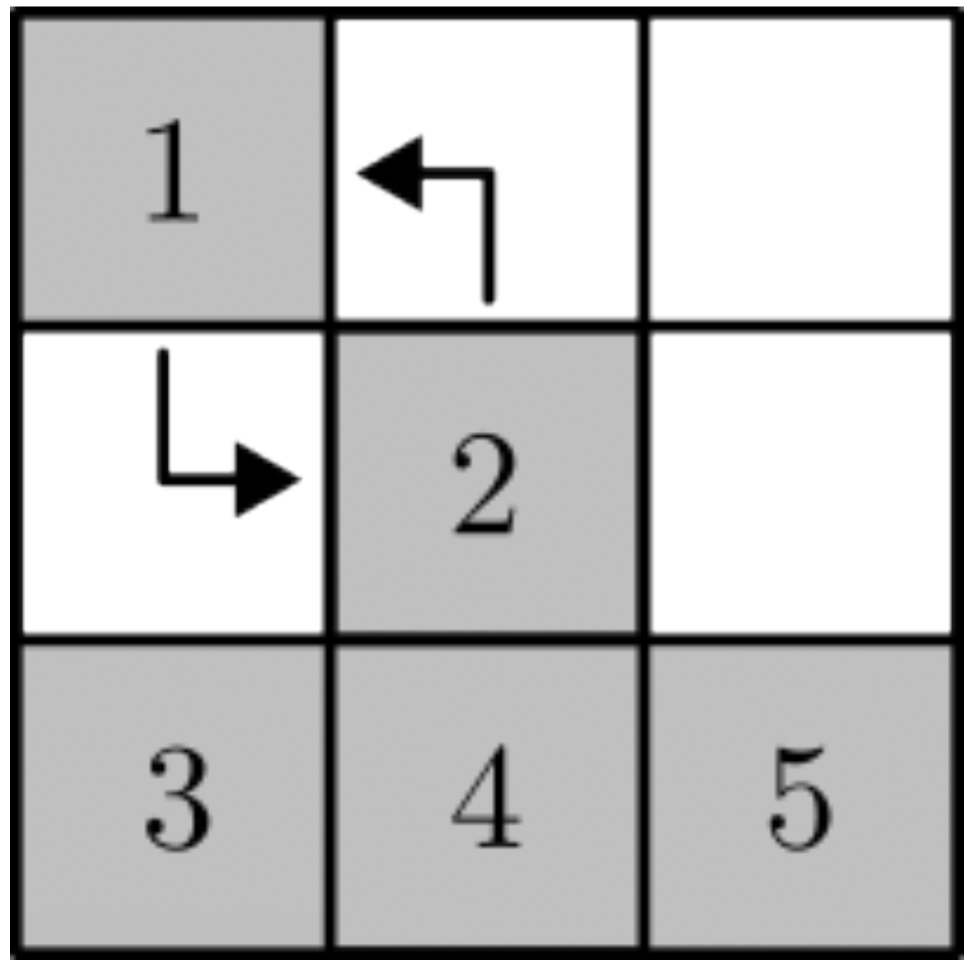

Proof Sketch

Mayer–Vietoris long exact sequence: Recovers the homology of $X = A_1 \cup A_2$ from the homology of A_1, A_2, and $A_1 \cap A_2$

$$\cdots \to H_{k+1}(X) \to H_k(A_1 \cap A_2) \to H_k(A_1) \oplus H_k(A_2) \to H_k(X) \to \cdots$$

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(Augmented) Mayer–Vietoris spectral sequence: Recovers the homology of $X = \cup_{i \in I} A_i$ from the homology of the $\cap_{j \in J \subseteq I} A_j$

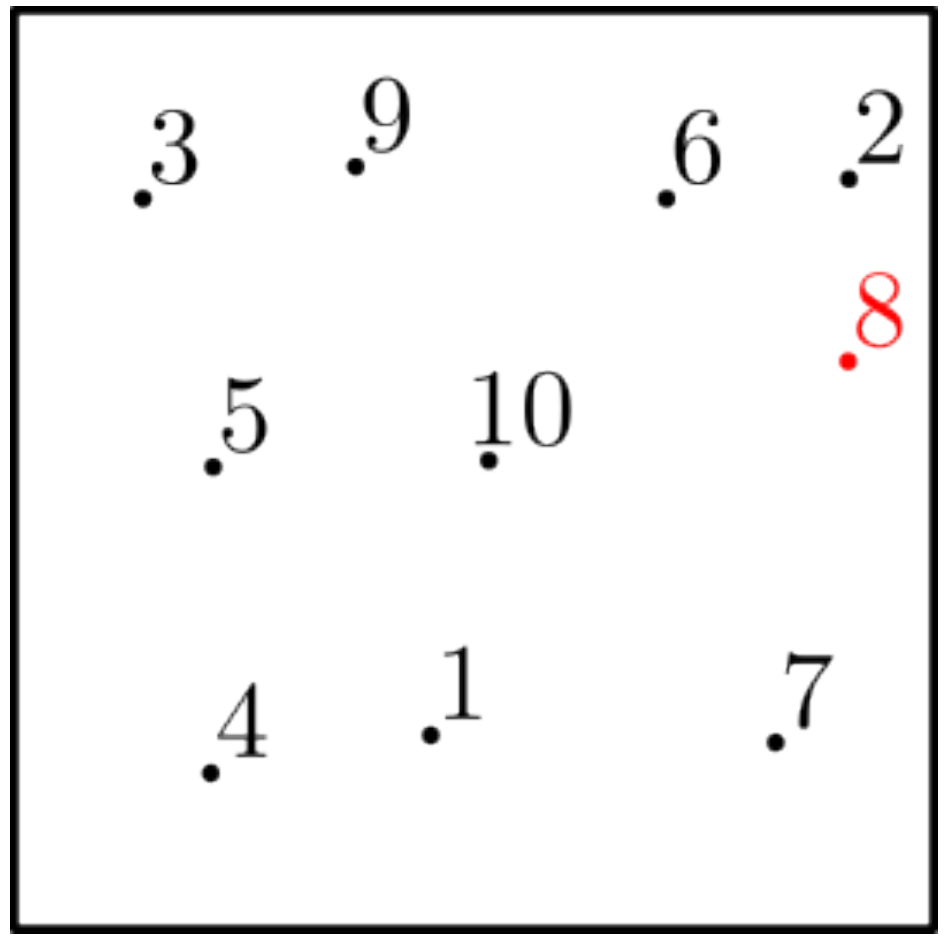
E^1-page

$H_2(X)$	$\oplus_{J \subset I, \lvert J\rvert=1} H_2(\cap_{j \in J} A_j)$	$\oplus_{J \subset I, \lvert J\rvert=2} H_2(\cap_{j \in J} A_j)$	$\oplus_{J \subset I, \lvert J\rvert=3} H_2(\cap_{j \in J} A_j)$
$H_1(X)$	$\oplus_{J \subset I, \lvert J\rvert=1} H_1(\cap_{j \in J} A_j)$	$\oplus_{J \subset I, \lvert J\rvert=2} H_1(\cap_{j \in J} A_j)$	$\oplus_{J \subset I, \lvert J\rvert=3} H_1(\cap_{j \in J} A_j)$
$H_0(X)$	$\oplus_{J \subset I, \lvert J\rvert=1} H_0(\cap_{j \in J} A_j)$	$\oplus_{J \subset I, \lvert J\rvert=2} H_0(\cap_{j \in J} A_j)$	$\oplus_{J \subset I, \lvert J\rvert=3} H_0(\cap_{j \in J} A_j)$
	0	1	2

Our Covers

$F_n(\mathbb{C}) = \bigcup_{i=1}^{n} F_n(\mathbb{C}; i)$

$F_n(\mathbb{C}; i)$ is the subspace of $F_n(\mathbb{C})$ in which no point is to the right of point i



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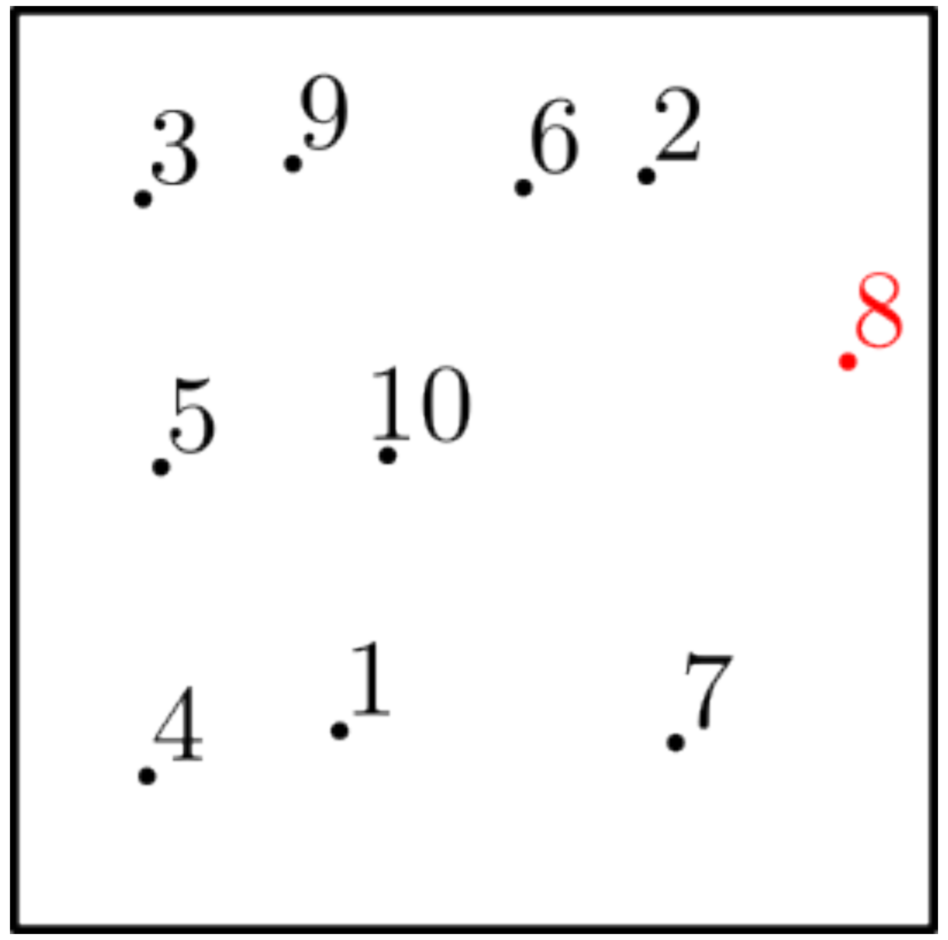

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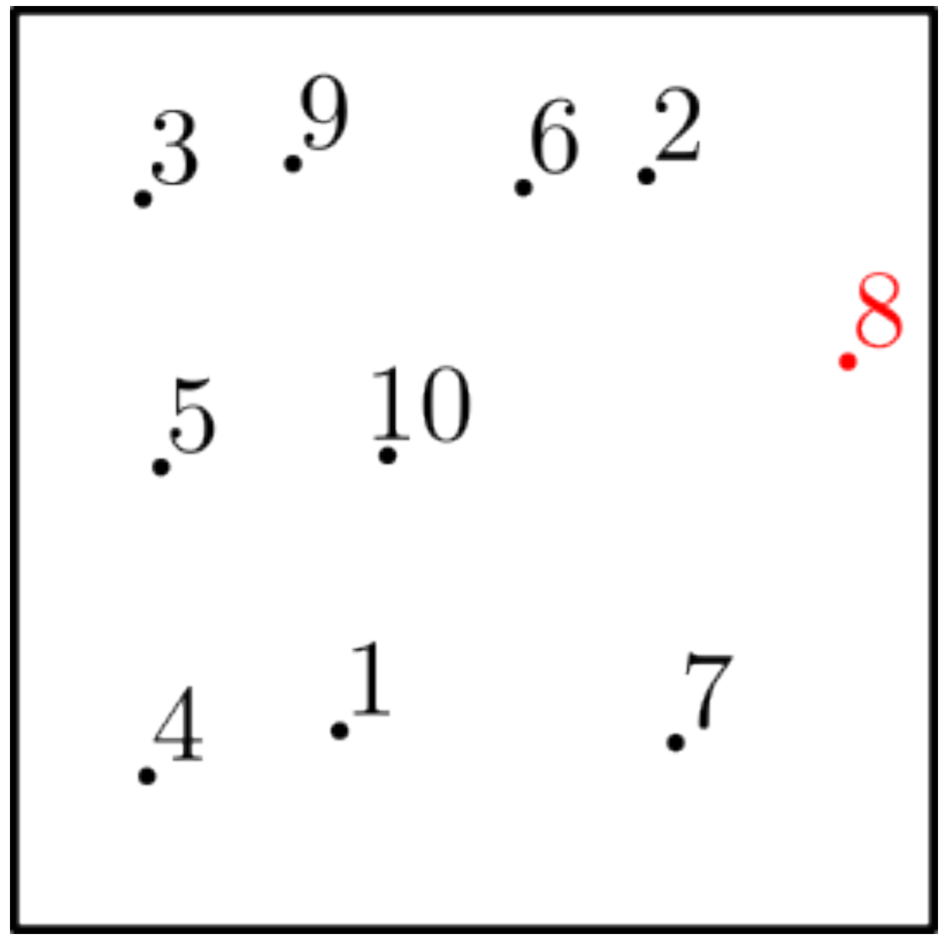

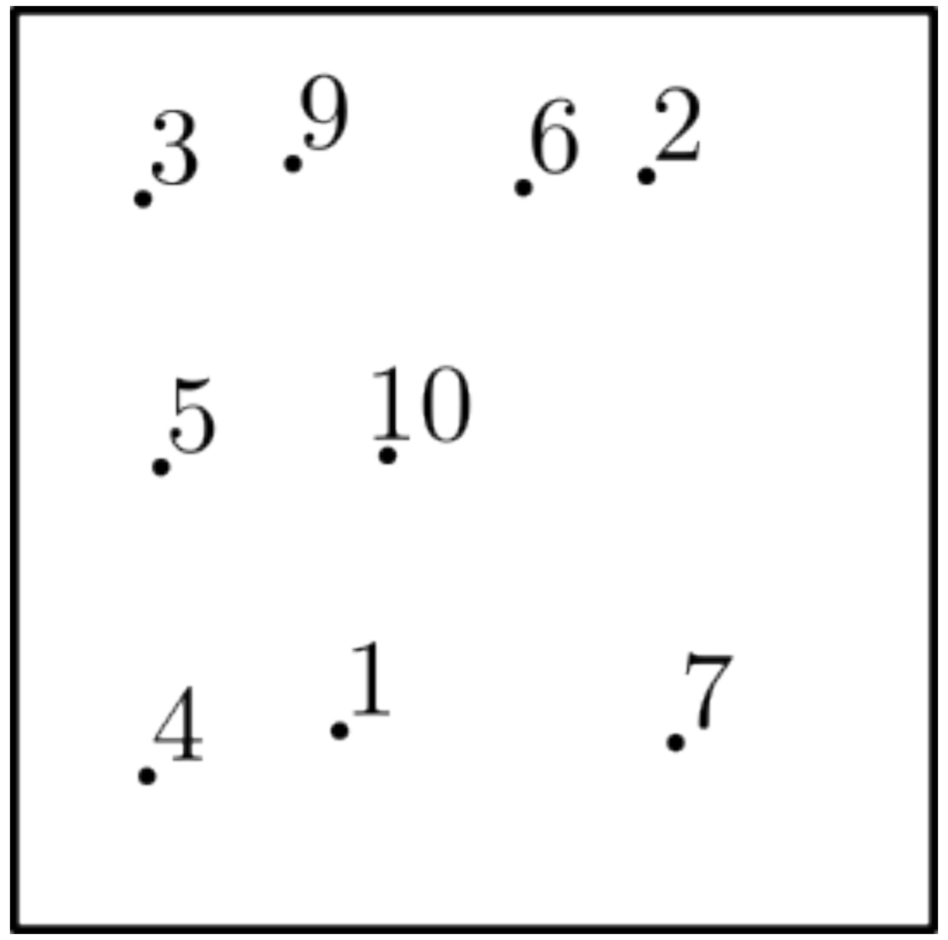

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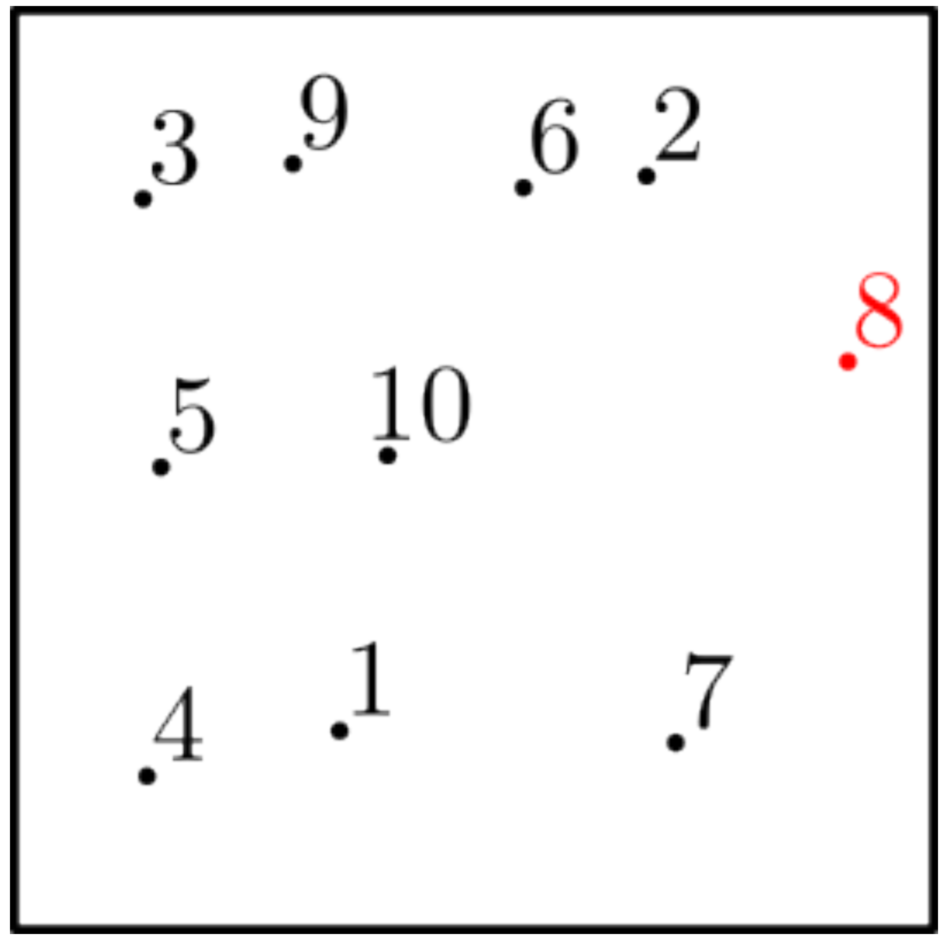
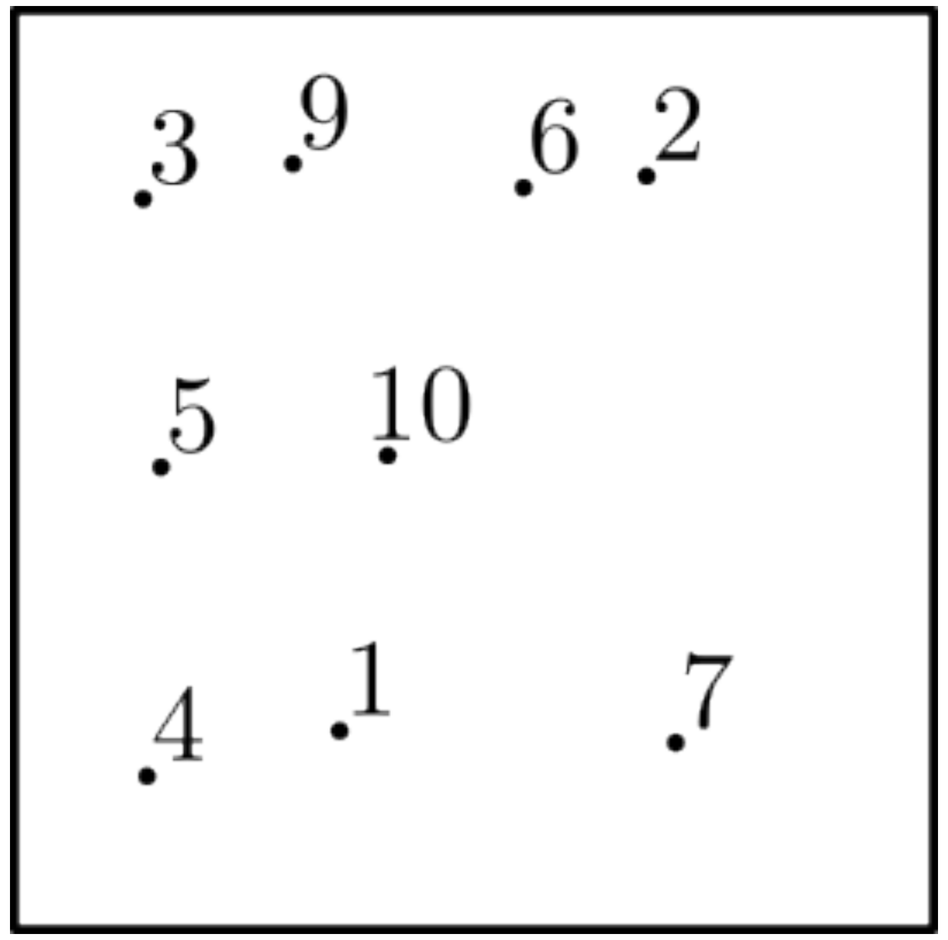
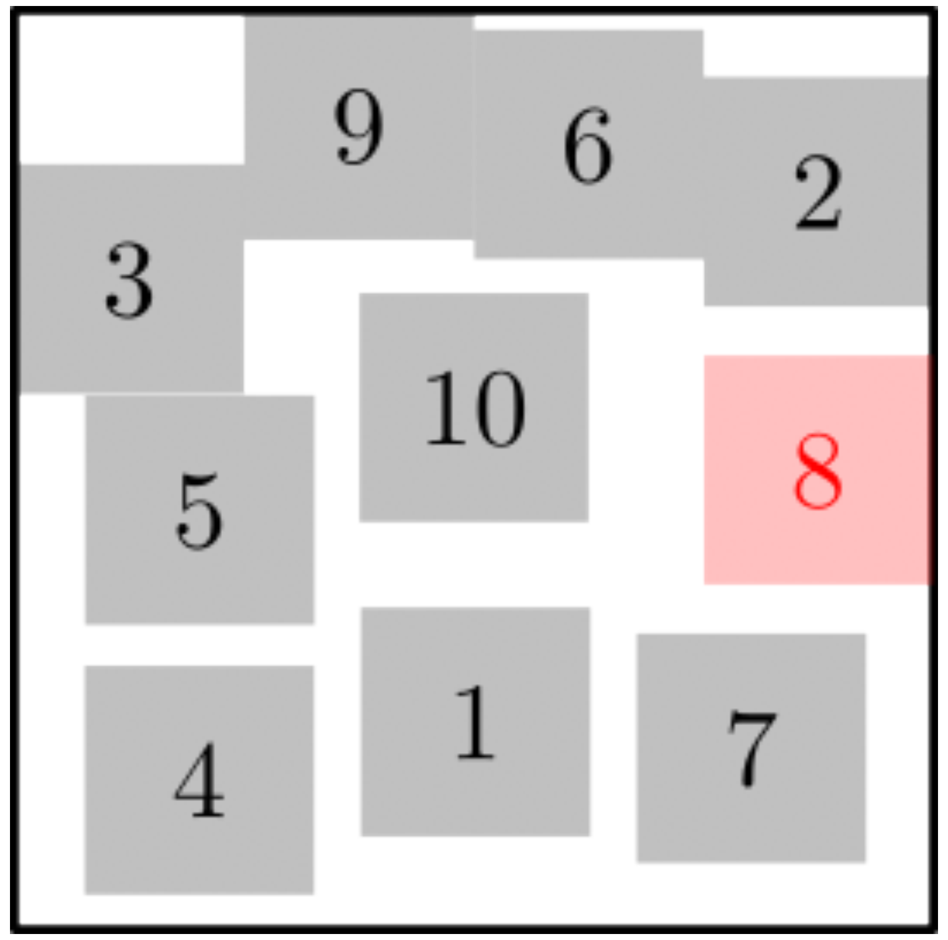




$SF_n(R_{w,h}) = \bigcup_{i=1}^{n} SF_n(R_{w,h}; i)$

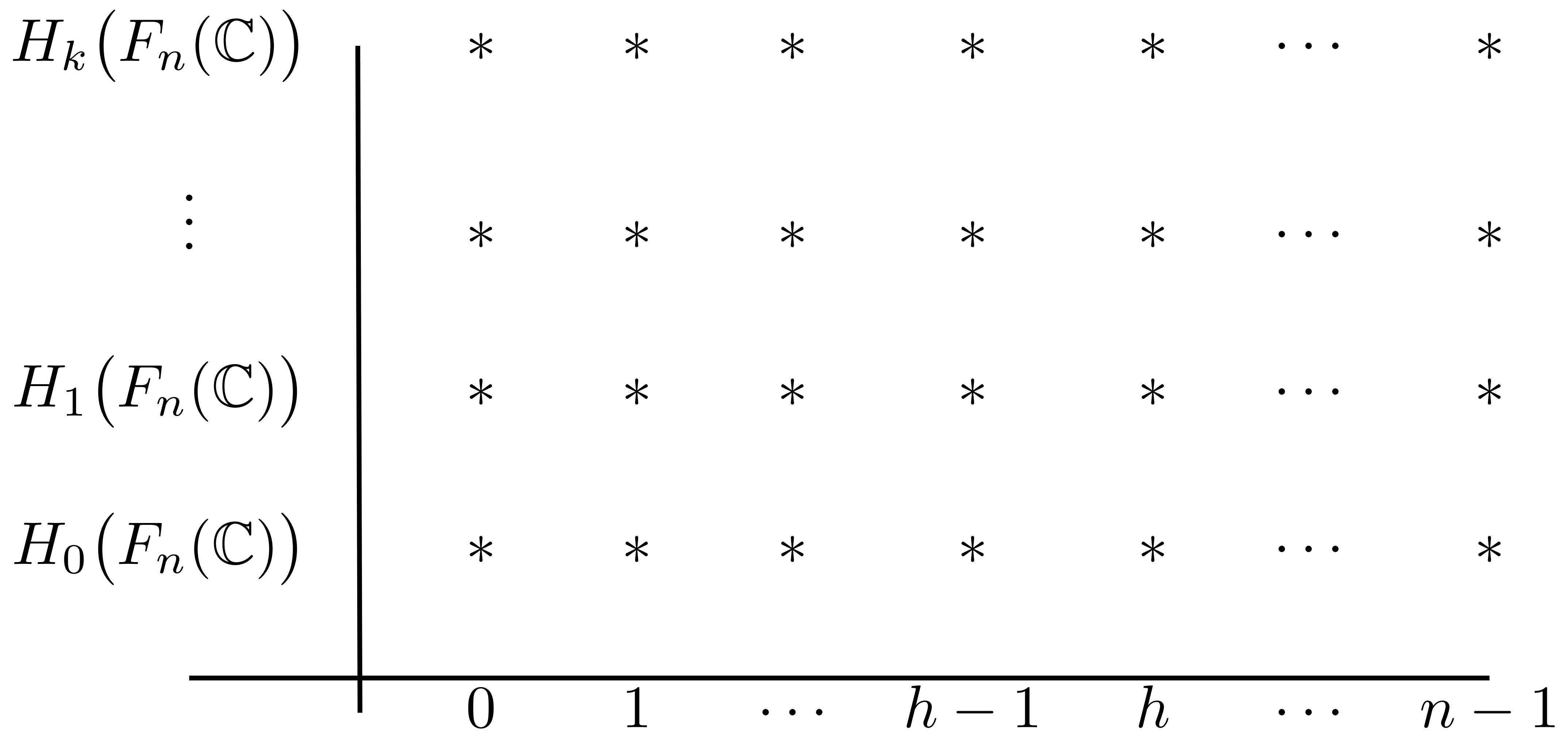
$SF_n(R_{w,h}; i)$ is the subspace of $SF_n(R_{w,h})$ in which no square is to the right of square i

$SF_n(R_{w,h}; i) \not\simeq SF_{n-1}(R_{w,h})$ in general



The $\mathbf{E^1}$-Pages

$$E^1_{p,q}\big[F_n(\mathbb{C})\big] \cong \bigoplus H_q\big(F_n(\mathbb{C}; i_0, \ldots, i_p)\big) \cong H_q\big(F_{n-p-1}(\mathbb{C})\big)$$



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$H_k\big(F_n(\mathbb{C})\big)$	$*$	$*$	$*$	$*$	$*$	$\cdots$	$*$
$\vdots$	$*$	$*$	$*$	$*$	$*$	$\cdots$	$*$
$H_1\big(F_n(\mathbb{C})\big)$	$*$	$*$	$*$	$*$	$*$	$\cdots$	$*$
$H_0\big(F_n(\mathbb{C})\big)$	$*$	$*$	$*$	$*$	$*$	$\cdots$	$*$
	0	1	$\cdots$	$h-1$	h	$\cdots$	$n-1$

$H_k\big(SF_n(R_{w,h})\big)$	$*$	$*$	$*$	$*$	0	$\cdots$	0
$\vdots$	$*$	$*$	$*$	$*$	0	$\cdots$	0
$H_1\big(SF_n(R_{w,h})\big)$	$*$	$*$	$*$	$*$	0	$\cdots$	0
$H_0\big(SF_n(R_{w,h})\big)$	$*$	$*$	$*$	$*$	0	$\cdots$	0
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$H_1\big(F_n(\mathbb{C})\big)$	$*$	$*$	$*$	$*$	$*$	$\cdots$	$*$
$H_0\big(F_n(\mathbb{C})\big)$	$*$	$*$	$*$	$*$	$*$	$\cdots$	$*$
	0	1	$\cdots$	$h-1$	h	$\cdots$	$n-1$

$H_k\big(SF_n(R_{w,h})\big)$	$*$	$*$	$*$	$*$	0	$\cdots$	0
$\vdots$	$*$	$*$	$*$	$*$	0	$\cdots$	0
$H_1\big(SF_n(R_{w,h})\big)$	$*$	$*$	$*$	$*$	0	$\cdots$	0
$H_0\big(SF_n(R_{w,h})\big)$	$*$	$*$	$*$	$*$	0	$\cdots$	0
	0	1	$\cdots$	$h-1$	h	$\cdots$	$n-1$

A Recursive Argument

Calculate differentials

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Prove $H_q\big(SF_n(R_{w,h}; i_0, \dots, i_p)\big) \cong H_q\big(F_n(\mathbb{C}; i_0, \dots, i_p)\big)$ in a range

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Do this via another Mayer–Vietoris spectral sequence

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Do this via another Mayer–Vietoris spectral sequence

Prevent turtles all the way down

Sharpness

Theorem (González–Kahle–W. 2026)

If $w \geq h \geq k+2$ and $wh - n \geq \max\left\{(k+1)(k+2), hk+2\right\}$, then

$$H_k\big(SF_n(R_{w,h})\big) \cong H_k\big(F_n(\mathbb{C})\big).$$

Theorem (González–Kahle–W. 2026)

If $wh - n < \frac{k(k+3)}{2}$, then

$$H_k\big(SF_n(R_{w,h})\big) \ncong H_k\big(F_n(\mathbb{C})\big).$$

New Prizes

Theorem (González–Kahle–W. 2026)

If $w, h \geq 3$ and $wh - n \geq 6$, then

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Prizes (W. 2026)

\$1,000 for a proof that

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\$100 for a proof that

$$H_1\big(SF_{12}(R_{4,4})\big) \cong H_1\big(F_{12}(\mathbb{C})\big).$$