

Name: _____

Score (Out of 5 points):

1. (5 points) For each of the following functions $f : X \rightarrow Y$ of topological spaces: If the function is continuous, write “**continuous**”. Otherwise, state an example of an open subset of Y whose preimage is not open, or a closed subset of Y whose preimage is not closed. **No further justification needed.**

Note: if the function is not continuous, you may receive partial credit for writing “**not continuous**” without writing a specific subset.

- (a) $X = (\mathbb{R}, \text{Euclidean}), Y = (\mathbb{R}, \text{cofinite}),$

$$\begin{aligned} f : X &\longrightarrow Y \\ x &\longmapsto \sin(x) \end{aligned}$$

- (b) $X = (\mathbb{R}, \text{cofinite}), Y = (\mathbb{R}, \text{Euclidean}),$

$$\begin{aligned} f : X &\longrightarrow Y \\ x &\longmapsto \sin(x) \end{aligned}$$

- (c) $X = (\mathbb{R} \setminus \{0\}, \text{indiscrete}), Y = (\mathbb{R}, \{U \mid 0 \in U\} \cup \{\emptyset\}),$

$$\begin{aligned} f : X &\longrightarrow Y \\ x &\longmapsto x \end{aligned}$$

$$(d) \ X = \left(\{0, 1\}, \left\{ \emptyset, \{1\}, \{0, 1\} \right\} \right), \ Y = \left(\{a, b, c\}, \left\{ \emptyset, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, b, c\} \right\} \right),$$

$$f : X \longrightarrow Y$$

$$0 \longmapsto a$$

$$1 \longmapsto c$$

$$(e) \ X = ([0, 1], \text{Euclidean}), \ Y = \left(\{a, b, c\}, \left\{ \emptyset, \{a\}, \{a, b\}, \{a, b, c\} \right\} \right),$$

$$f : X \longrightarrow Y$$

$$x \longmapsto \begin{cases} a, & x \in \left[0, \frac{1}{2}\right) \\ b, & x \in \left[\frac{1}{2}, 1\right) \\ c, & x = 1 \end{cases}$$