Storage of a strong magnetic field below the solar convection zone

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Abstract. We discuss the storage of a strong magnetic field for two different field configurations: magnetic flux tubes and magnetic layers. In the first case, we are mainly interested in the influence of radiative energy transport and meridional circulation. Radiative inflow of heat leads to enhanced buoyancy which causes the flux tube to move upwards and leave the storage region. In the second case, we discuss possible equilibrium configurations and the conditions under which these equilibria can be reached starting from a non-equilibrium state. In the solar overshoot region we obtain solutions similar to the case of flux tubes, for which the magnetic curvature force is compensated by the Coriolis force.

1. Introduction

Simulations of erupting flux tubes show that essential properties of sunspot groups require an initial field strength of about $10\,\mathrm{T}$ ($10^5\,\mathrm{Gauss}$) for the flux tubes at the base of the convection zone (D'Silva & Choudhuri 1993, Fan et al. 1993, Caligari et al. 1995). Such a strong magnetic field can only be stably stored in a subadiabatically stratified layer below the solar convection zone proper (Ferriz-Mas & Schüssler 1993, 1995). In the first part of this contribution, we discuss the consequences of radiative heating of flux tubes stored in a region of convective overshoot.

On the other hand, it is not clear whether the field is in fact in the form of flux tubes from the beginning. A conceivable alternative is a more or less homogeneous layer of magnetic field, which generates flux tubes only after becoming unstable. Equilibrium configurations for flux tubes and the conditions under which these can be reached have been considered by Moreno-Insertis et al. (1992). In the second part, we give a similar analysis for magnetic layers extended between pole and equator.

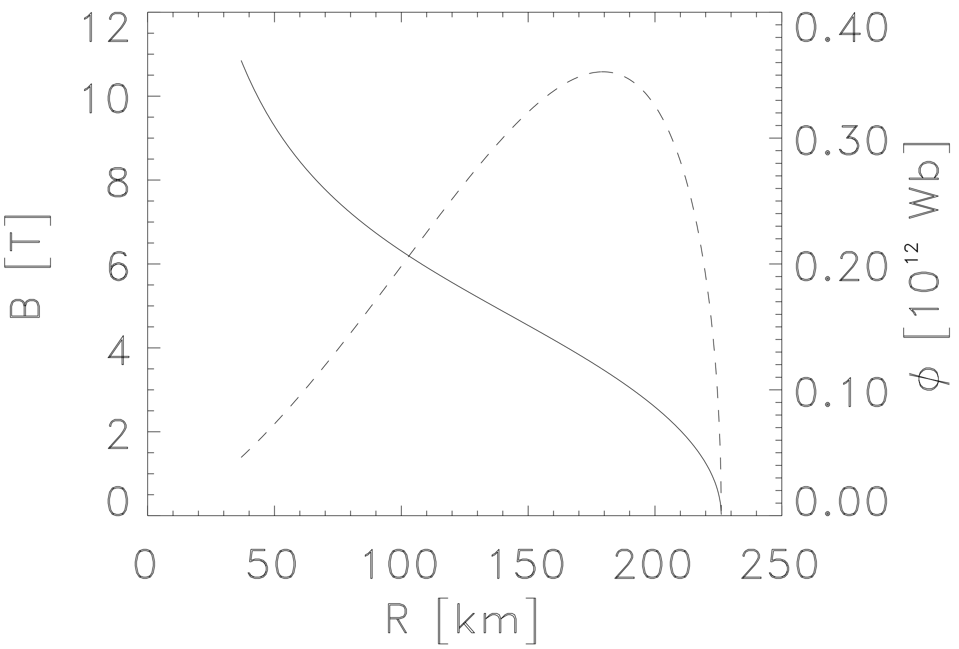

Figure 1. Field strength (solid line) and magnetic flux (dashed line) as a function of tube radius for flux tubes in mechanical and thermal equilibrium (including radiative energy exchange and a meridional circulation) at a latitude of 20° and for $\delta = -10^{-6}$. The magnetic flux of a single tube is more than two magnitudes smaller than the flux of a typical sunspot (10^{14} Wb). Storage of tubes with such a flux requires a stronger subadiabaticity of the stratification, $\delta \lesssim -10^{-4}$.

2. Radiative heating of flux tubes

We consider the storage of a strong magnetic field in form of flux tubes in the solar overshoot region, including the effects of radiative energy exchange and of the drag force due to a meridional circulation. The basic assumptions are the thin flux tube approximation and axial symmetry (flux rings). In the case without meridional flow in the storage region, mechanical equilibrium of a flux tube requires the following conditions (Moreno-Insertis et al. 1992):

- compensation of the magnetic curvature force by the Coriolis force (both forces are perpendicular to the axis of rotation),

- neutral buoyancy (no other force in radial direction).

The condition of neutral buoyancy can only be achieved in a subadiabatic stratification. In the case with a meridional flow (equatorward in the overshoot region), the drag force has to be included in the force balance. As a result, mechanical equilibria with finite buoyancy are possible; these become important if the effects of radiative heating are included. The two main causes for radiative heating of flux tubes in the solar overshoot region are:

- radiative inflow of heat due to a lower temperature inside a neutrally buoyant flux tube (this effect is negligible for flux tubes with a radius $\gtrsim 1$ Mm) (Moreno-Insertis 1983),

- non-vanishing divergence of the external radiative energy flux leading to a heating rate of about $3\,\mathrm{W/m^3}$ in the overshoot region and in the lower part of the convection zone; the origin of this effect lies in the transition between radiative and convective energy transport (Fan & Fisher 1996).

The radiative heating of the flux tube leads to enhanced buoyancy and disturbs the mechanical equilibrium. The resulting rise velocity of the flux tube depends strongly on the superadiabaticity $\delta = \nabla - \nabla_{\mathrm{ad}}$ and can be estimated roughly as $v \approx 10^{-5}/|\delta|\,\mathrm{m{\cdot}s^{-1}}$ (for $\delta < 0$). For typical values of $\delta \approx -10^{-6}$ in the overshoot layer, the flux tube escapes from the storage region in a time scale of months. This rough estimate is confirmed by more detailed flux tube simulations. A new equilibrium can only be found if mechanical equilibrium and thermal equilibrium are fulfilled simultaneously. Because a drag force is required for mechanical equilibrium with finite buoyancy, such equilibria are restricted to the case with a meridional flow and to flux tubes with sufficiently small radius (see Fig. 1), unless the stratification is rather strongly subadiabatic. This is mainly due to the fact that the drag force is only relevant for small tube radius. As a consequence, the storage of magnetic flux tubes with the flux of a typical sunspot (10^{14} Wb) for time scales comparable to the solar cycle requires a superadiabaticity $\delta \lesssim -10^{-4}$.

3. Equilibrium models for magnetic layers

As an alternative to (or a predecessor of) a flux tube configuration, we have studied the equilibrium of an axially symmetric magnetic layer of toroidal field. We allow for a stationary toroidal flow (in addition to rotation) and consider the limit $\beta = p_{\mathrm{gas}}/p_{\mathrm{mag}} \gg 1$; the latter condition leads to small disturbances of pressure (p_1) and density (ϱ_1) compared to the background stratification. This allows us to reduce the partial differential equations of stationary MHD to ordinary differential equations in the latitudinal direction, which can be solved easily. A particular solution is specified by prescribing the pressure disturbance as a function of the radial coordinate for a fixed latitude. For a symmetric magnetic layer between pole and equator we have chosen $p_1 = -B^2/(2\,\mu_0)$ at a latitude of 45° (the numerical simulations described in the next section confirm the validity of this choice). Two kinds of solutions are found: 1) equilibria with a similar force balance as in the case a flux tube, i.e., the magnetic curvature force is balanced by the Coriolis force; 2) solutions for which the latitudinal component of the curvature force is compensated by latitudinal gradient of the total pressure (gas plus magnetic), while in the radial direction we have a balance of pressure gradient, curvature force and gravity; this kind of force equilibrium can only arise in a strongly subadiabatic stratification. The main features of such a solution are shown in Fig. 2.

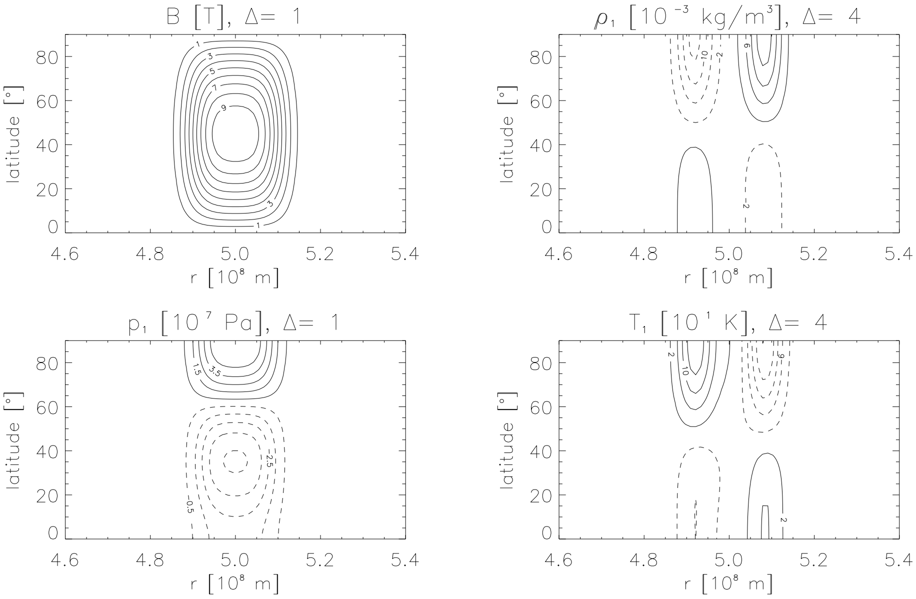

Figure 2. Magnetic field strength (B) and the disturbances of gas pressure (p_1), density (ρ_1), and temperature (T_1) for a magnetic layer without toroidal flows. Δ denotes the interval between two contour levels, dashed curves indicate negative values. The latitudinal component of the magnetic curvature force is compensated by a latitudinal pressure gradient. The force balance in the radial direction requires a density disturbance that is nearly proportional to the radial gradient of the total pressure disturbance, $p_1 + B^2/(2\,\mu_0)$.

4. MHD-Simulations

The equilibrium configurations for magnetic layers discussed above are of practical interest only if they can be reached starting from a non-equilibrium state. In order to study this process, its time scale and the dependence on the superadiabaticity of the background stratification, we have performed numerical simulations of the axisymmetric time-dependent MHD equations using the Versatile Advection Code[1] (Tóth 1996). In order to deal with the small Mach numbers resulting from the large values of β, we have used a numerical scheme with a very small numerical viscosity (MacCormack scheme); we have also rewritten the MHD equations in terms of perturbations with respect to the (hydrostatic) background stratification. In addition, the small value of $|\delta|$ in the overshoot region and lower convection zone forces us to decompose the pressure equation into an adiabatic part and a term that treats the deviations caused by the non-adiabatic stratification. This allows us to directly use the superadiabaticity of

[1] http://www.phys.uu.nl/~toth

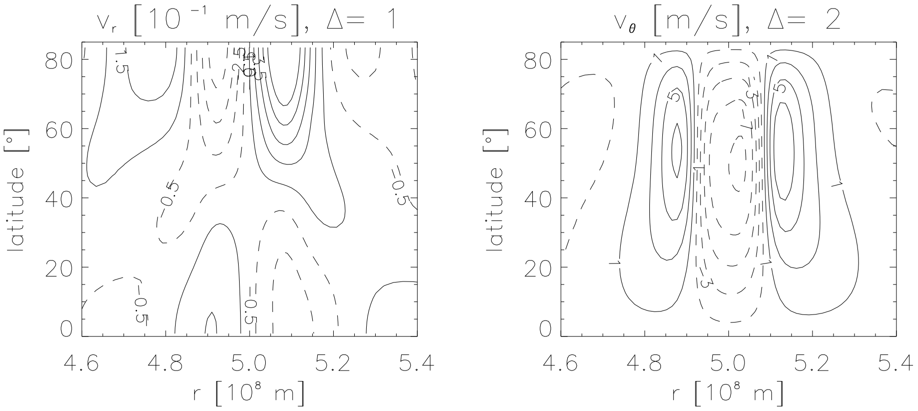

Figure 3. Approach towards an equilibrium in a strongly subadiabatic stratification ($\delta_0 = -0.1$). Shown are the radial component (left panel) and the latitudinal component (right panel) of the velocity after $5 \cdot 10^4$ s. Positive values (solid curves) correspond to upflows (left panel) and equatorward flows (right panel), respectively. The initial magnetic field is the same as in Fig. 2. The latitudinal velocity is driven by the latitudinal component of the magnetic curvature force. Symmetric boundary conditions at pole and equator lead to the radial velocity pattern with up- and downflows near the pole and equator.

the background stratification, δ_0, without taking numerical derivatives, whose error would be comparable to the effects of the small superadiabaticity itself.

The results of the simulation can be summarized as follows: The magnetic curvature force drives a two-cell flow (cf. Fig. 3), which establishes an equilibrium configuration. Two processes are decisive: firstly, the transport of angular momentum by the meridional motion leads to a toroidal flow, which tends to balance the curvature force by means of the Coriolis force (with a time scale similar to the rotation period); secondly, the radial motions in a subadiabatic stratification generate temperature fluctuations that build up a latitudinal pressure gradient (with time scale $\tau \propto |\delta|^{-1/2}$). These two processes lead to the different types of equilibrium configurations. In the first case, the magnetic curvature force is completely compensated by the Coriolis force; only a small disturbance of the stratification is required in order to balance the total pressure gradient. In contrast, the second case requires larger disturbances of the stratification (cf. Fig. 2). The magnitude of the superadiabaticity of the background stratification determines, which of the two processes controls the development of the equilibrium configuration:

- $\delta_0 \approx -0.1$: Mechanical equilibrium is established in $\approx 10^5$ s, the influence of a toroidal flow can be neglected;

- $\delta_0 \approx -10^{-3}$: Mechanical equilibrium is established in $\approx 10^6$ s, the influence of the toroidal flow is comparable to the effect of the latitudinal pressure gradient;

- $\delta_0 > -10^{-3}$: The equilibrium is determined by the toroidal flow, the disturbances p_1 and ϱ_1 are negligible.

Consequently, the compensation of the magnetic curvature force by a latitudinal pressure gradient is only important in the radiative region ($\delta \approx -0.2$). In the overshoot region ($\delta \approx -10^{-6}$), the storage requires a rather strong toroidal flow (100 m/s), which is comparable to the velocity differences of the differential rotation itself. This equilibrium configuration is comparable to the flux tube equilibria discussed in Moreno-Insertis et al. (1992). A fragmentation of such a layer would result in flux tubes with similar properties. Only if viscosity prevents equilibria with a strong toroidal flow, pressure gradients could be important in the overshoot layer. The smoothing of temperature fluctuations by convective diffusion would then be crucial.

References

Caligari P., Moreno-Insertis F. & Schüssler M., 1995, ApJ, 441, 886

D'Silva, S., Choudhuri, A.R., 1993, A&A, 272, 621

Fan Y., Fisher G.H. & DeLuca E.E., 1993, ApJ, 405, 390

Fan Y. & Fisher G.H., 1996, Solar Phys., 166, 17

Ferriz-Mas A. & Schüssler M., 1993, Geophys. Astrophys. Fluid Dyn., 72, 209

Ferriz-Mas A. & Schüssler M., 1995, Geophys. Astrophys. Fluid Dyn., 81, 233

Moreno-Insertis F., 1983, A&A, 122, 241

Moreno-Insertis F., Schüssler M. & Ferriz-Mas A., 1992, A&A, 264, 686

Tóth G., 1996, Astrophys. Lett. & Comm., 34, 245