# NONLINEAR MHD SIMULATIONS OF WAVE DISSIPATION IN FLUX TUBES

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**Abstract.** The phase-mixing and resonant dissipation of Alfvén waves is studied in both the 'closed' magnetic loops and the 'open' coronal holes observed in the hot solar corona. The resulting energy transfer from large to small length scales contributes to the heating of these magnetic structures. The nonlinear simulations show that the periodically varying shear flows that occur in the resonant layers are *unstable*. In coronal holes, the *phase-mixing* of running Alfvén waves is *speeded-up* by the 'flaring-out' of the magnetic field lines in the lower chromosphere.

**Key words:** nonlinear MHD, resonant dissipation, phase-mixing

## 1. Introduction

Resonant dissipation of Alfvén waves in solar coronal loops is one of the most promising candidates for the explanation of the million degree temperatures observed in the solar corona. As a matter of fact, numerous analytical and numerical studies have shown that this heating mechanism can be very efficient and the time scales on which it operates turn out to be acceptable, *i.e.*, much shorter than the typical life-time of a coronal loop (about one day). Recent observations of Alfvén waves in the chromosphere (Bocchialini & Koutchmy, 1995) and the corona (Cheng et al., 1979; O'Shea et al., 1996) seem to support the wave heating theory. However, the theoretical predictions are mostly based on *linear* MHD calculations. Poedts (1996) has demonstrated that the dynamics of the shear flow in the resonant layers is very nonlinear for coronal parameter values and the heating mechanism might turn out to be less efficient when these nonlinear effects are taken into account. In the present paper, it will be shown that the shear flow in the resonant layers is *unstable* and the resonant layers are partly destroyed by this instability.

The phase-mixing mechanism is much less efficient in coronal holes. This is due to the fact that these 'open' magnetic structures do not form resonant cavities and the waves run out of the structure as they are 'phase-mixed'. In straight flux tube model calculations, the phase-mixing mechanism turns out to be too slow to dissipate enough energy to explain the observed temperatures. In the present paper, it will be shown that the flaring-out of the magnetic field lines in the photospheric and lower-chromospheric regions *speeds-up* the phase-mixing process.

The time scale of resistive diffusion, $\tau_{\text{diff}} = l^2/\eta$ (with $l$ a typical length scale and $\eta$ the magnetic diffusivity), is extremely long in the hot coronal plasma. Therefore, any heating mechanism is, in fact, a mechanism to create small length scales in order to reduce the dissipation time scales considerably. In the 'phase-mixing' and 'resonant absorption' mechanisms, small length scales $l \sim (\eta + \nu)^{1/3}$ (with $\nu$ the viscosity) are created. Linear MHD studies (see *e.g.* Sakurai & Granik, 1984) have shown that resulting time-scales for both resonant dissipation and phase-mixing,

$$\tau_{\text{RA}}, \tau_{\text{mix}} \sim (\eta + \nu)^{-1/3} \, (\omega_A')^{-2/3}. \tag{1}$$

Both time scales are much shorter than $\tau_{\text{diff}}$ for typical coronal parameters ($\eta, \nu \ll 1$). In the above equation, $\omega_A'$ denotes the derivative of the local Alfvén frequency with respect to the spatial direction normal to the magnetic surfaces. Hence, the heating time scale is proportional to the cube root of both the Reynolds number and the magnetic Reynolds number *and* affected by the inhomogeneities in the plasma. However, the question on the efficiency of these wave heating mechanisms and their contribution to coronal heating requires time-dependent, nonlinear, 3D computer simulations as will be convincingly demonstrated in the present paper.

## 2. Resonant Dissipation in Coronal Loops

The 'closed' magnetic loops observed in the solar corona can operate as resonant cavities. The *standing waves* that can be excited in such structures behave spatially singular at the positions where their frequency matches the local Alfvén frequency, $\omega_A \equiv \mathbf{k}\cdot\mathbf{B}/\sqrt{\mu_0\rho}$. As a model for a large aspect-ratio coronal loop we adopt a cylindrical flux tube with a sheared magnetic field and excited *side-ways* by incident waves. The resulting dynamics is described by the MHD equations, which can be written in dimensionless form as:

$$\rho \frac{\partial \mathbf{V}}{\partial t} = -\mathbf{V}\cdot\nabla\mathbf{V} - \nabla p + (\nabla\times\mathbf{B})\times\mathbf{B}/\mu_0, \tag{2}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{V}\times\mathbf{B}) - \nabla\times(\eta\nabla\times\mathbf{B}), \tag{3}$$

$$\begin{aligned} \frac{\partial p}{\partial t} &= -\nabla\cdot(p\mathbf{V}) - (\gamma-1)p\,\nabla\cdot\mathbf{V} \\ &\quad + (\gamma-1)\eta(\nabla\times\mathbf{B})^2/\mu_0, \end{aligned} \tag{4}$$

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{V}), \tag{5}$$

$$\nabla \cdot \mathbf{B} = 0. \tag{6}$$

Here, $\rho$, $p$, $\mathbf{V}$, $\mathbf{B}$, and $\eta$ denote the plasma density, the plasma pressure, the velocity field, the magnetic field, and the magnetic diffusivity, respectively. The ratio of specific heats, $\gamma$, is assumed to be 5/3. In the dimensionless set of Eqs. (2)–(6) lengths are normalized to the characteristic scale of the cylindrical system, *i.e.*, the plasma radius ($a$), and magnetic fields, velocities, and times are normalized to the initial longitudinal magnetic field on the axis ($B_0$), to the Alfvén velocity ($V_A = B_0/\sqrt{\mu_0 \rho}$), and the Alfvén crossing time ($\tau_A = a/V_A$), respectively. Note that gravity has been ignored and in the present section the density is assumed to be constant. This yields a good approximation for Alfvén waves (which do not involve a density perturbation anyway) and saves an enormous amount of CPU time (Poedts, 1996).

The external region is not included in the computational domain and the incident waves are taken into account through the boundary conditions at the loop surface ($r = 1$) where a periodic driver of the following form is specified:

$$v_r(\varphi, z; t) = A e^{i m_d \varphi + i \frac{2\pi}{L} n_d z} \sin(\omega_d t), \tag{7}$$

in cylindrical coordinates $(r, \varphi, z)$. Here, $m_d$, $n_d$, $A$, and $\omega_d$ denote the azimuthal and longitudinal wave numbers, the amplitude, and the frequency of the driver (incident wave), respectively. In the longitudinal direction we used simple periodic boundary conditions. The effect of anchoring of the field lines in the dense photosphere will be studied in a forthcoming paper.

Consider an initial plasma column of lenght $L = 40\pi$, with a constant density $\rho_0$, a constant $B_{0z}$-component, and a parabolic current density $J_{0z} = 0.2(1 - r^2)$. We first simulated the effect of the side-ways excitation of a loop by a simple wave in the *linear* regime. After some time, depending on the magnetic Reynolds number, a steady state sets in in which the energy supplied by the external driver is exactly balanced by the Ohmic dissipation in the resonant layer (Poedts & Kerner, 1992). In this *linear* steady state, the loop dynamics is characterized by a shear flow in a narrow resonant layer as shown in Fig. 1

The amplitude of this *periodically varying* flow in the resonance layer depends on the magnetic Reynolds number and on the inhomogeneity of the background equilibrium:

$$v_\perp \sim A\, \eta^{-1/3} \left(\omega_A'\right)^{-2/3} \sin(\omega_d t). \tag{8}$$

Heyvaerts and Priest (1983) have shown analytically that such shear flows are subject to Kelvin-Helmholtz instabilities for typical coronal loop parameter values and that the growth rate is larger than $2\omega_d$, which is required

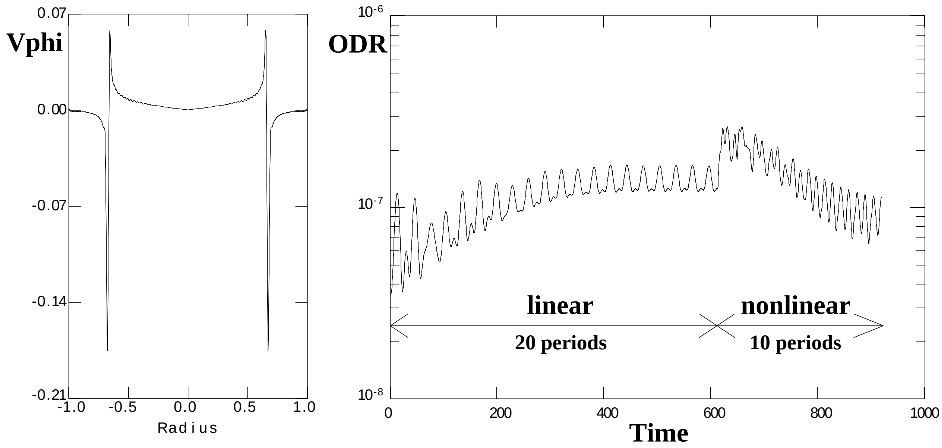


*Figure 1.* (left) Snapshot of the $V_\varphi$-component of the velocity after 80 driving periods for $A = 0.01\, V_A$, $m_d = 2$, $n_d = 1$, and $\omega_d = 0.205$ yielding a resonance at $r = 0.671$ in this *linear* MHD context; and (right) dimensionless Ohmic dissipation rate versus time for a linear MHD calculation (first part) that is continued nonlinearly (second part) after $20\, \mathrm{P}_d$ with a resolution of $256 \times 32 \times 1$ (in $(r, \varphi, z)$, with the $n$'s chosen such that $m/n = 2$), $\eta = 64 \times 10^{-7}$ ($R_m = 1.5625 \times 10^5$).

because the flow changes periodically. Our numerical simulations indeed confirm this and, in addition, reveal the effect of this instability on the efficiency of the heating mechanism and on the localization of the heat deposition in the loop.

Fig. 2 demonstrates the deformation of the resonant layer as a result of the Kelvin-Helmholtz instability of the shear flow. In Fig. 2 snapshots are shown of $(\nabla \times \mathbf{v})_z$. The eight vortices present in the linear steady state, due of the choice $m_d = 2$, are deformed. Also, a complex dynamical behaviour develops in the nonlinear continuation of the linear steady state, with vortex splitting occuring frequently.

The heating rate turns out to increase first and then decrease again to a level *below* the linear heating rate (see Fig. 1b). So the energy already present in the resonant layer dissipates faster in nonlinear MHD due to the energy cascade to smaller length scales, as is clear from Fig. 2, but pumping in new energy turns out to be less efficient due to the partial destruction of the resonance by the Kelvin-Helmholtz instability of the shear flow.

Snapshots of the electric current density look very similar to Fig. 2 so that the Ohmic dissipation is not limited to a narrow resonant layer, as in linear MHD results, but instead spreads out over the loop volume, even for a simple incident wave with a single driving frequency!

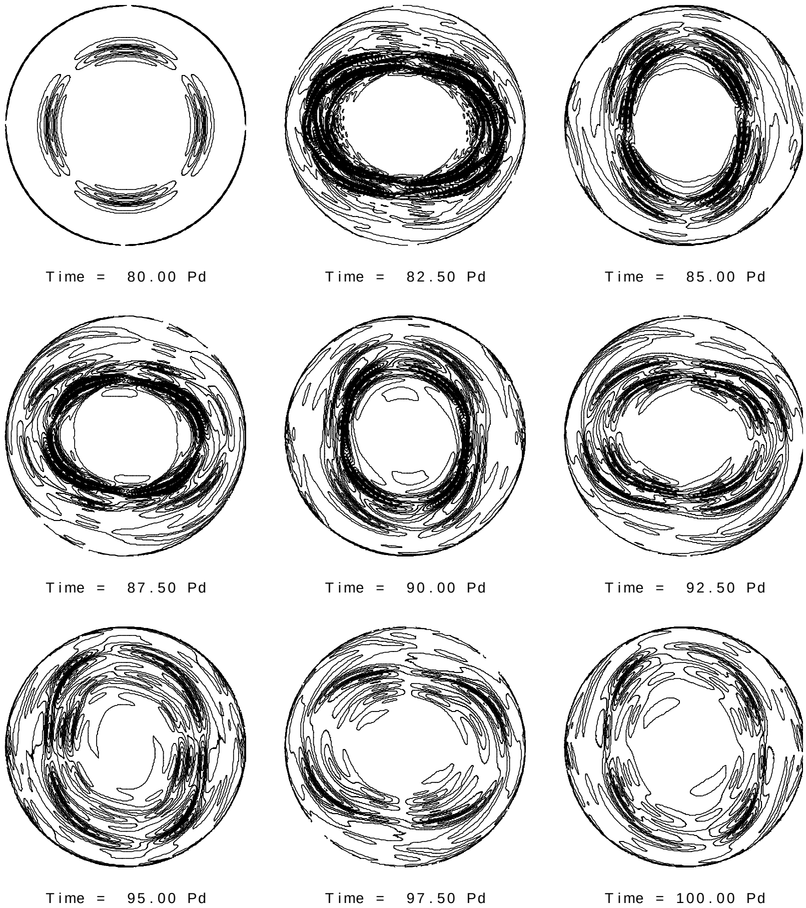


*Figure 2.* Snapshots of $(\nabla \times \mathbf{v})_z$ for a nonlinear continuation of a linear run after the 'steady state' is obtained for the same parameters as in Fig. 1 ($R_m = 1.5625 \times 10^5$). The top-left picture shows the linear steady state.

## 3. Alfvén Waves in Coronal Holes

Coronal holes are 'open' magnetic structures that do not normally operate as resonant cavities. Hence, the Alfvén waves, which can easily be excited, in such structures are *running* waves. When the local Alfvén velocity is different on different magnetic flux surfaces, these waves are phase-mixed as they propagate in vertical direction. Analytical studies (*e.g.*, Sakurai and Granik (1984)) have shown that the time scale for this process depends in exactly the same way on $\eta$, $\nu$, and the background inhomogeneity as the time scale for resonant absorption (see Eq. 1).

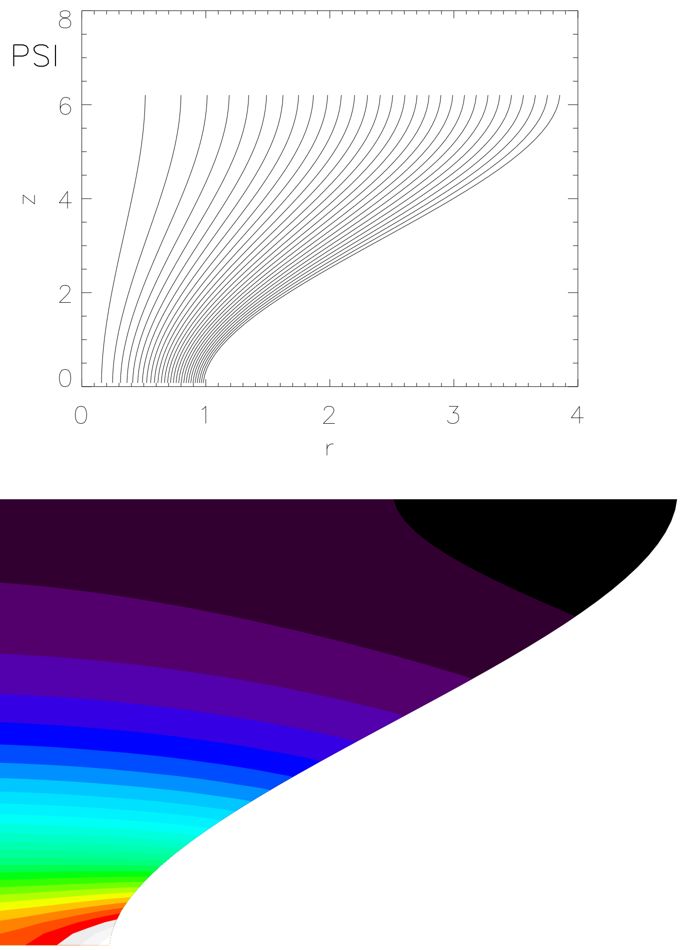


*Figure 3.* The magnetic field lines (top) in a coronal hole model 'flaring-out' by a factor of four, and contour plot of the corresponding local Alfvén velocity (bottom, dark indicating low and light indicating high values).

The temporal evolution of the phase-mixing of Alfvén waves in coronal holes was simulated numerically by means of the VAC code which solves Eqs. (2)–(6) in conservative form (Tóth & Odstrčil, 1996). The coronal holes are modeled by cylindrical axisymmetric configurations. The initial equilibria are obtained by means of PARIS (Beliën et al., 1996). In Fig. 3 the local Alfvén velocity, $V_A \equiv B/\sqrt{\mu_0 \rho}$, is plotted in the initial state. Of course, the Alfvén velocity reduces by a factor of sixteen from bottom to top of the flaring tube because of flux conservation ($r^2 B =$ constant). It is also clear, however, that strong radial gradients occur in the magnetic field strength,

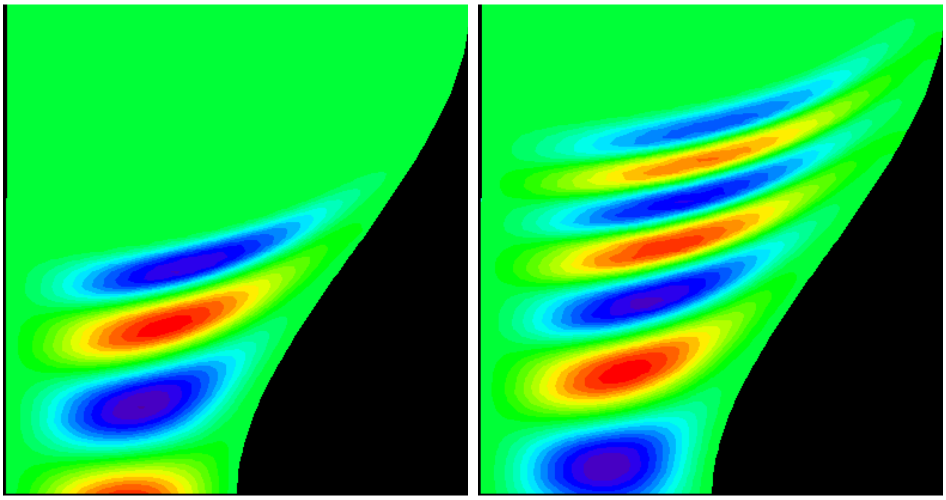

*Figure 4.* Two snapshots at $t = 0.565\, L_z/V_{A0}$ (left) and at $t = 1.131\, L_z/V_{A0}$ (right) of the $B_\varphi$-component of a running Alfvén wave in a coronal hole characterized by a 'flaring-out' of the field lines by a factor of two ($V_{A0} \equiv V_A(r = z = t = 0)$).

and therefore also in the Alfvén velocity, especially at the bottom of the coronal hole where the field lines start to flare out. The resulting *decrease of the phase-mixing time*, through the term $(\omega'_A)^{-2/3}$ in Eq. 1, can be considerable, provided the wavelengths of the waves that pass trough this region are not too long.

Alfvén waves have been excited in such structures by means of a driver of the form

$$v_\varphi(r;t) = 16\, A\, r^2(1-r)^2 \sin(\omega_d t), \tag{9}$$

at the botom of the coronal hole. A typical 2D example of the resulting phase-mixing is shown in Fig. 4. The speed-up of the phase-mixing depends on the wavelengths of the excited Alfvén waves and on the extension of the region in which the field lines flare out. In the results shown in Fig. 4, the density was taken constant so that the observed phase-mixing is entirely due to the effect of the flaring-out of the flux tube.

## 4. Concluding Remarks

In conclusion, the dynamics of the resonant absorption of Alfvén waves is very nonlinear indeed. The shear flow in the resonant layers that form when coronal loops are excited externally by incident waves with frequencies in the range of the ideal MHD continuous spectrum, turns out to be unstable in nonlinear MHD. The instability leads to an energy cascade to smaller length scales and faster dissipation of the energy stored in the resonant layers. However, the Kelvin-Helmholtz instability also deforms the resonances and 'pumping in' new energy becomes less efficient after a while.

In solar coronal holes, the 'flaring-out' of the magnetic field lines due to the decrease of the external pressure in the lower chromosphere yields a very inhomogeneous magnetic pressure in that region. When this is taken into account, the phase-mixing of running Alfvén waves in such magnetic structures can be speeded-up considerably. It is not sure, however, whether this speed-up is enough for a substantial contribution of phase-mixing to the heating of coronal holes.

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