Numerical simulations of stellar winds

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Abstract. We discuss steady-state transonic outflows obtained by direct numerical solution of the hydrodynamic and magnetohydrodynamic equations. We make use of the Versatile Advection Code, a software package for solving systems of (hyperbolic) partial differential equations. We model thermally and magneto-centrifugally driven stellar outflows as generalizations of the well-known Parker and Weber-Davis wind solutions. To obtain steady-state solutions efficiently, we exploit fully implicit time stepping.

Wind solutions containing both a 'wind' and a 'dead' zone are presented. We emphasize the boundary conditions imposed at the stellar surface. For axisymmetric wind solutions, we use the knowledge of the flux functions to verify the numerical solutions.

1. Axisymmetric transonic MHD flows

We use cylindrical coordinates (R, Z, φ) and ideal MHD. For $\partial\varphi = 0$ and $\partial t = 0$, mass and flux conservation lead to a poloidal stream function $\chi(R, Z)$ and a poloidal flux function $\psi(R, Z)$ with $\chi(\psi)$. All axisymmetric stationary ideal MHD equilibria are derivable from a single variational principle $\delta L = \delta \int \mathcal{L} dV = 0$ with Lagrangian density (Goedbloed et al., 1998):

$$\mathcal{L}(M^2, \psi, \nabla\psi; R, Z) = \frac{1}{2R^2}(1 - M^2) \mid \nabla\psi \mid^2 - \frac{\Pi_1}{M^2} + \frac{\Pi_2}{\gamma M^{2\gamma}} - \frac{\Pi_3}{1 - M^2}$$

The minimizing Euler-Lagrange equations need to be solved simultaneously for the flux function $\psi(R, Z)$ and the squared poloidal Alfvén Mach number $M^2(R, Z) \equiv \rho v_p^2 / B_p^2 = \chi'^2/\rho$. They describe force balance across poloidal streamlines and energy conservation along them. Integrating the axisymmetric, stationary ideal MHD equations, one obtains five flux functions. These enter in the profile expressions Π_i. The flux functions are the Bernoulli function ($\sim$ energy) $H \equiv \frac{1}{2}v^2 + (\gamma/(\gamma - 1))\, p/\rho - GM_g/r - R\Omega v_\varphi$ the derivative of the stream function $\chi' \equiv \partial\chi/\partial\psi$, the entropy S, the total specific angular momentum $L \equiv Rv_\varphi - RB_\varphi/\chi'$, and the derivative of the electric field potential $\Omega \equiv R^{-1}(v_\varphi - (\chi'/\rho)B_\varphi)$. By an appropriate choice of the *free* flux functions, we can construct axisymmetric, stationary flows. Self-similar solutions can be analysed further by analytic means. General solutions require a numerical solution procedure.

2. Versatile Advection Code

VAC (Tóth, 1996) is a general purpose software package for solving a conservative system of partial differential equations with additional non-hyperbolic source terms. We solve the system on a finite volume structured grid. Particular systems available are the hydrodynamic (Euler and Navier-Stokes) and the magnetohydrodynamic (ideal and resistive) equations. The source code is dimension independent, so it can solve 1D, 2D, 2.5D, and 3D problems. A modular approach ensures the compatibility between the different code segments. As a result, several spatial and temporal (explicit, semi-implicit and fully implicit) discretizations are applicable to all physics modules.

3. Magnetized winds containing a 'dead' zone

With VAC, we can calculate steady-state transonic solutions to the MHD equations which mimick thermally and/or magneto-centrifugally driven stellar outflows. Under polytropic $p \sim \rho^\gamma$ conditions, analytic solutions are available for equatorial outflow from a magnetized, rotating star. In Keppens & Goedbloed (1998), we explain how we start from a hydrodynamic, non-rotating, spherically symmetric, isothermal outflow (Parker wind), and relax all assumptions to allow for magnetic fields, rotation and polytropic flows. The resulting Weber-Davis solution has slow r_s and fast r_f critical points, and an Alfvén point r_A where the radial velocity v_r equals the radial Alfvén speed A_r. Using modern numerical schemes, we generate Weber-Davis solutions. The positions of and velocities at these specific points can be verified to agree perfectly with analytic solutions *a posteriori*.

Extrapolating the Weber-Davis solution out of the equatorial plane yields polytropic, axisymmetric and transonic outflow from magnetized, rotating stars. One obtains critical surfaces, and a continuous acceleration from subslow flow at the surface to superfast flow at large distances. Poloidal streamlines of constant $\chi(R, Z)$ and poloidal field lines of constant $\psi(R, Z)$ are parallel. Specific angular momentum and all other flux functions are conserved along them.

By a suitable initialization of the time-marching procedure to obtain steady-state stellar wind solutions, we can find magnetized, rotating winds containing both a 'wind' and a 'dead' zone. Boundary conditions at the stellar surface impose the mass flux and ensure the correct rotational coupling of velocity and magnetic field. We let the density and all magnetic field components adjust freely at the base. This allows for the simultaneous and self-consistent modeling of both open and closed field

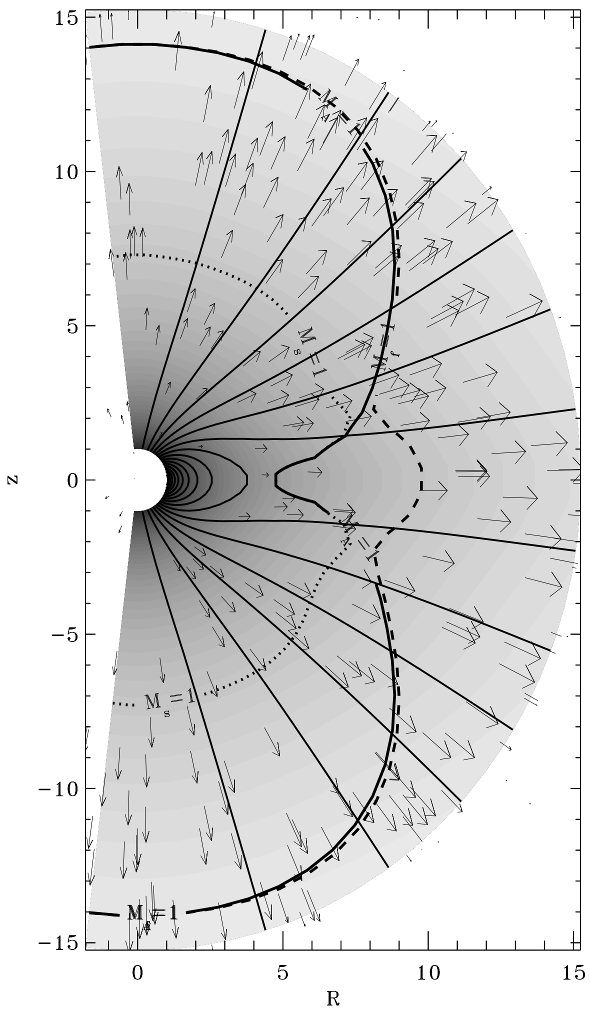

Figure 1. Axisymmetric magnetized wind containing a 'wind' and a 'dead' zone. Shown are the poloidal magnetic field lines and the poloidal flow field as vectors. Indicated are the three critical surfaces where $M_s^p = 1$ (dotted), $M_A^p = 1$ (solid line), and $M_f^p = 1$ (dashed). The density structure is shown in a grey scaling.

line regions, thereby extending models presented by Sakurai (1985). Figure 1 shows the overall solution, described in detail in Keppens & Goedbloed (1998). Our method easily allows us to incorporate a latitudinal dependence of the stellar rotation rate, or the magnetic field strength. The latitudinal variation of the density and the velocity can be compared with Ulysses (SWOOPS) measurements (McComas et al., 1998). Clearly, the density is higher about the ecliptic and there is a decrease in wind speed associated with the equatorial 'dead' zone. Of particular interest is the way in which the outflow accelerates to superfast flow, thereby defining and crossing the critical surfaces where the poloidal flow exceeds the poloidal slow, Alfvén, and fast velocities.

4. Conclusions

A single variational principle governs all axisymmetric transonic MHD flows. While specific classes of self-similar flows can be studied analytically, general flow solutions are to be obtained by large-scale 3D MHD codes. VAC is such a general-purpose code, running on vector and parallel supercomputers, and we demonstrated its capacity to model stellar winds. So far, we presented steady-state transonic outflows in spherical and axisymmetry. In Keppens & Goedbloed (1998), we modeled: (i) isothermal, spherically symmetric Parker winds; (ii) polytropic, spherically symmetric Parker winds; (iii) polytropic, rotating Parker winds for the equatorial plane; (iv) Weber-Davis magnetized, polytropic, rotating winds for the equatorial plane; and finally axisymmetric, polytropic, rotating 2D winds, both (v) unmagnetized and magnetized, without (vi) and with (vii) a 'dead' zone. We can now meaningfully explore time-dependent 3D wind modeling. Extensions of such thermal and magneto-centrifugal MHD wind models could explore additional wave-driven phenomena, which presumably play a role in fast solar polar wind streams.

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