Non-linear dynamics of Kelvin-Helmholtz unstable magnetized jets

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Introduction

We present a numerical study of the Kelvin-Helmholtz instability for 2D and 3D shear flow configurations in compressible magnetohydrodynamics.

The 2D results consider an initial magnetic field aligned with the shear flow, either unidirectional or changing sign at the interface. In the latter case, the initial current sheet gets amplified by the vortex flow and can become unstable to tearing instabilities forming magnetic islands.

The 3D simulations consider shear flow in a cylindrical magnetized jet. The growth of linear perturbations at specified poloidal and axial mode numbers lead to induced secondary Kelvin-Helmholtz instabilities at higher mode numbers. The initially weak magnetic field becomes locally dominant in the non-linear dynamics before and during saturation. Thereby, it controls the jet deformation.

Kelvin-Helmholtz instability

The Kelvin-Helmholtz (KH) instability occurs at the interface between two fluids or plasmas in the presence of shear flow, e.g. causing waves on the sea surface due to wind.

In 2D [1], we compared three cases: (i) a purely hydrodynamic case; (ii) a 'uniform' magnetohydrodynamic – MHD – case where the magnetic field at $t = 0$ is $B_0\hat{x}$; and (iii) a 'reversed' MHD case where the magnetic field at $t = 0$ is $-B_0\hat{x}$ for $y > 0$ and $B_0\hat{x}$ for $y < 0$. We applied a shear velocity in the x-direction with amplitude V_0, i.e. $v_x = V_0 \tanh(y/a)$. We perturbed this configuration with a velocity component $\delta v_y = v_{y0} \sin(k_x x) \exp\left[-y^2/\sigma^2\right]$, perpendicular to the background shear velocity.

The initial phase of the evolution is one of exponential growth. We determined the growth rate as a function of wavenumber, sound Mach number, and Alfvén Mach number (see Fig. 1). While a uniform magnetic field stabilizes, a reversed field acts to destabilize.

Secondary tearing instabilities

When looking at time series of the density for a uniform and a reversed field case, it can be seen how the density pattern becomes controlled by the magnetic field. Field lines are dragged away from the center of the layer and stretched by the vortical flow. Regions of enhanced density contain narrow, depleted lanes which correspond to compressed field lines. In the reversed field case, isolated island structures become evident within the region of enhanced density. The anti-parallel field lines are pushed together there, which is tearing unstable in resistive MHD. Magnetic islands form and this process continues along the current sheet. The magnetic field lines are shown in Fig. 2 at a time following the non-linear saturation stage of the KH instability. The 2D current-vortex sheet evolution subsequently transits to magnetically modified turbulence.

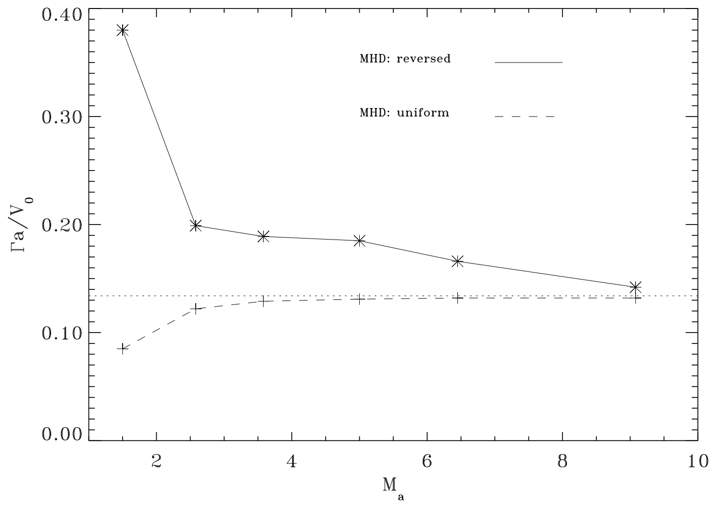

Figure 1: *We show the dependence of the growth rate* Γ *on the Alfvén Mach number* $M_a = V_0/(B_0/\sqrt{\rho_0})$*. For low magnetic fields, both uniform (+) and reversed (*∗*) case reach the HD limit (dotted line).*

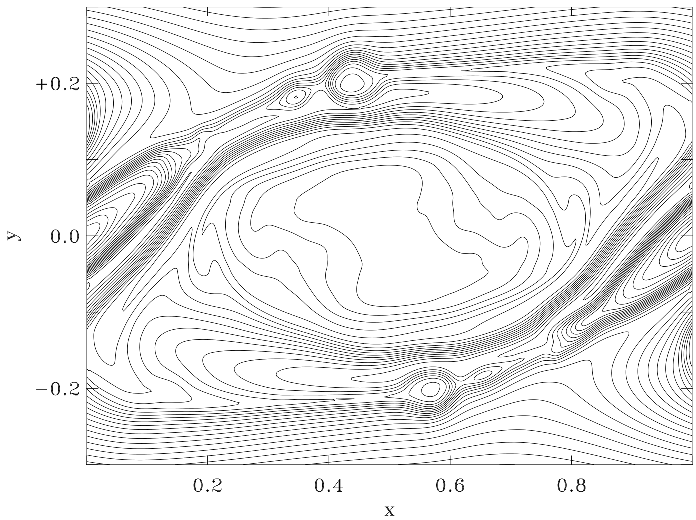

Figure 2: *The magnetic field structure in an evolved current-vortex sheet, showing islands that have formed through tearing, as the primary Kelvin-Helmholtz instability develops.*

3D magnetized jets

In 3D [2], we considered a jet-like flow in a uniform magnetic field. Using (R, φ, Z) cylindrical coordinates, we set up a flow field

$$v_Z(R; t = 0) = V_0 \tanh \frac{R - R_{jet}}{a}. \qquad (1)$$

The jet surface $R = R_{jet}$ is easily identified as the $v_Z = 0$ isosurface.

Taking $V_0 = 0.645$ and $B_0 = 0.129$, the 3D configuration has identical properties as the 2D case shown above. We perturbed the jet by a radial velocity profile

$$v_R(R, \varphi, Z; t = 0) = \Delta v_R \exp - \left(\frac{R - R_{jet}}{4a}\right)^2 \cos(m\varphi) \sin \frac{n2\pi Z}{L}. \tag{2}$$

We imposed a perturbation at specific mode numbers $m = 1$ and $n = 1$.

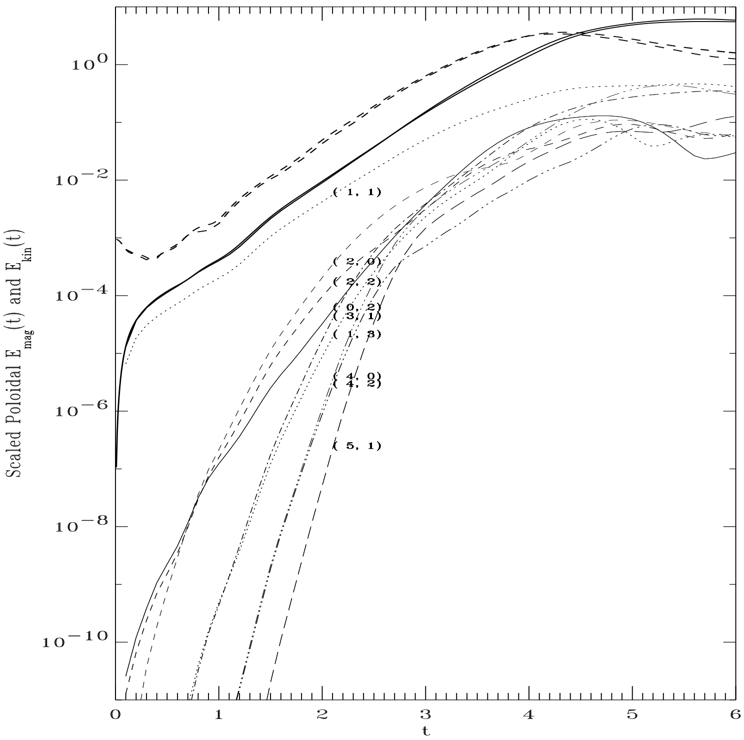

Figure 3: *The scaled poloidal kinetic energy (thick dashed) indicates the nonlinear saturation of the KH instability at time $t = 4$. The thick solid line is the poloidal magnetic energy, with the contributions of the most important mode number pairs plotted.*

When determining the contributions to the poloidal magnetic energy of the most dominant wave numbers (Fig. 3), we found that the $(1, 1)$ perturbations induce $(0, 2)$, $(2, 2)$, and $(2, 0)$ modes, in that order. These couple and the ordening of their amplitudes reverse at $t \simeq 0.75$. At saturation $t = 4$, the three most important mode number pairs are the excited $(1, 1)$ and the induced $(3, 1)$ and $(0, 2)$ modes. The evolution of the individual linear modes was most easily extracted from an independent calculation of the same jet configuration using a 3D pseudo-spectral code. Both simulations agreed perfectly: the time histories of the poloidal kinetic and magnetic energies from each run are shown and essentially overlap in Fig. 3.

At the particular aspect ratio $L/R_{jet} = 2$, the $(m, n) = (1, 1)$ kink perturbation of the jet leads to a secondary KH instability, characterized by an axial $n = 2$ wave number, along top and bottom of the jet. The induced poloidal pressure gradient triggers this instability. The magnetic field gets compressed in narrow sheets that intersect $(1, 1)$ compression zones, and in 3D fibrils due to these secondary KH flows. These, in turn, cause an $m = 3$ deformation of the jet surface (see Fig. 4).

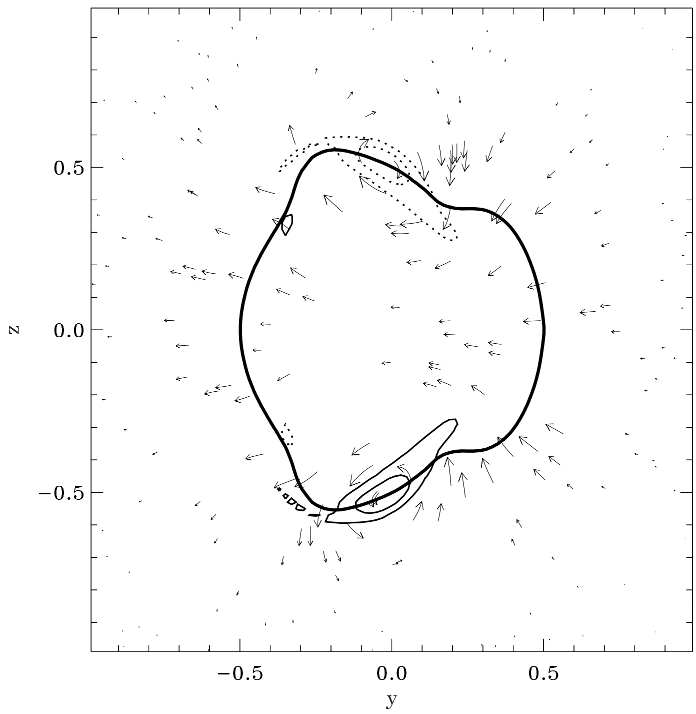

Figure 4: *A cut midway through the jet at saturation, demonstrating the $m = 3$ deformation of the jet surface (solid line), and the secondary vortical flows on the top and bottom of the jet, swirling around a 3D fibril of strong magnetic field.*

Conclusions

We investigated the role of the magnetic field in the dynamics of Kelvin-Helmholtz unstable shear flows. Our numerical study made use of the *Versatile Advection Code*, a software package designed to solve general systems of conservation laws. VAC is available via registration at `http://www.phys.uu.nl/~toth`.

In our 2D study [1], we showed that when the initial field changes sign at the interface, the growth rate is higher than in the pure hydrodynamic case. Such an initial current sheet gets distorted and amplified by the vortex flow, and tearing instabilities form chains of magnetic islands.

In a 3D study [2] of a magnetized jet, even a weak magnetic field ultimately governs the density structure in the jet flow and its deformation.

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[1] R. Keppens, G. Tóth, R.H.J. Westermann, and J.P. Goedbloed, "Growth and saturation of the Kelvin-Helmholtz instability with parallel and anti-parallel magnetic fields", *J. Plasma Phys.* **61**(1), 1–19 (1999).

[2] R. Keppens and G. Tóth, "Non-linear dynamics of Kelvin-Helmholtz unstable magnetized jets: three-dimensional effects", *Physics of Plasmas* **6**(5), 1461–1469 (1999).