The evolution of the compressible magnetized wake

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Abstract

A compressible magnetohydrodynamic study of the magnetized wake is presented. Such wakes possess both magnetic and velocity shear and are suitable idealizations for studying e.g. the heliospheric current sheet. Both linear and nonlinear results place particular emphasis on the superAlfvénic speed flow situation. In this flow dominated regime, unstable sinuous and varicose modes are found, with the kink (sinuous) mode growing the fastest. We quantify the influence of compressibility by varying the sonic Mach number. In general, the growth rates of these modes decrease with increasing Mach number. We analyze the nonlinear development of the most unstable, high speed wake configurations, demonstrating mode steepening into fast magnetosonic shocks and an eventual relaxation to a more laminar flow regime.

The magnetized wake

The magnetized wake is a magnetohydrodynamic equilibrium configuration where a planar, 2D wake flow given by

$$\mathbf{v} = [1 - \text{sech } y]\hat{e}_x,$$

is embedded in a 3D, force-free, sheared magnetic field of constant magnitude. With

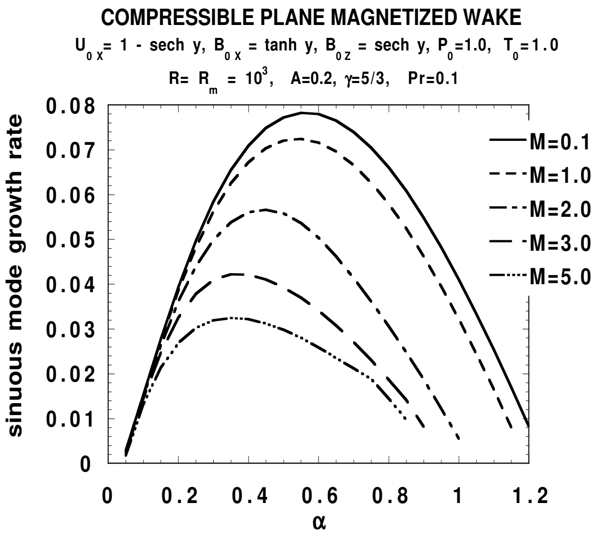
$$B_{0x}(y) = \tanh y \quad \text{and} \quad B_{0z}(y) = \text{sech } y,$$

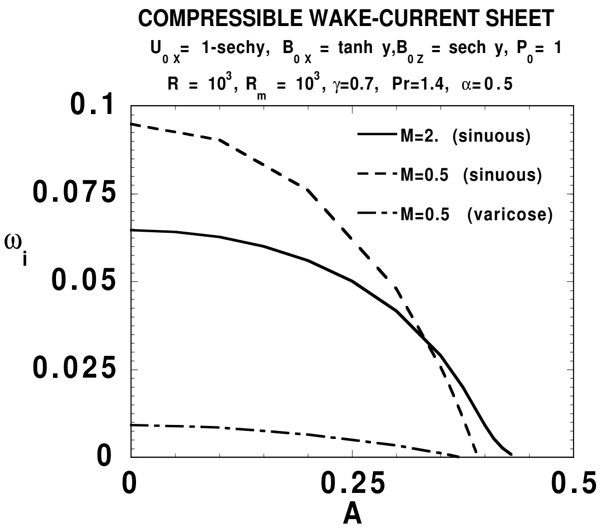
the magnetic field rotates about the y-axis in an out-of-plane fashion from parallel alignment with the flow in the far upper halfspace $y > 0$ to antiparallel alignment with the flow in the far lower halfspace $y < 0$. The co-spatial magnetic and velocity shear mimicks conditions found at distinct locations throughout the heliosphere.

The stability and nonlinear evolution of similar multiple shear systems has widely been studied. Ref. [1] demonstrated the existence of two ideally unstable modes for a purely planar jet flow $\mathbf{v} = \text{sech } y\hat{e}_x$ in a neutral sheet $B_{0x}(y) = \tanh y$ in the incompressible, flow dominated regime. These varicose (sausage) and sinuous (kink) modes again appear as the primary Kelvin-Helmholtz type instabilities for the superAlfvénic wake. Ref. [2] looked at 2D compressible current-vortex configurations where $\mathbf{v} = \tanh y\hat{e}_x$ and $\mathbf{B} = \tanh y\hat{e}_x$ and found subsonic, superAlfvénic scenarios where both mode types appear. The Kelvin-Helmholtz instabilities thereby led to induced tearing behaviour in the further nonlinear evolution.

Influence of compressibility

We restrict the attention to cases where the Alfvén number A (ratio Alfvén speed to flow speed) is below unity. For low sonic Mach numbers, both sausage and kink instabilities can be found, recovering the incompressible limits. The figure below [left] shows how the dispersion curve relating growth rate to the streamwise (x-direction) wavenumber α varies when the Mach number is raised. The sinuous mode shown here for $A = 0.2$ (or Alfvén Mach number $M_A = 5$) survives for supersonic wakes, although its growth rate decreases substantially. The varicose mode gets fully stabilized for increasing compressibility and has a smaller growth rate.





At right in the figure, we depict the growth rates for the unstable modes of a subsonic $M = 0.5$ and a supersonic $M = 2$ wake when going from unmagnetized $A = 0$ to weakly magnetized – hence flow-dominated, superAlfvénic – cases. Note how both sinuous and varicose modes exist for $M = 0.5$, but only the sinuous mode is present for $M = 2$. Increasing the magnetic field strength acts to suppress all modes.

The dispersion relations were obtained numerically using a compressible magnetohydrodynamic extension of the SPECLS code, which uses Chebyshev polynomials to describe the cross-stream y-variation of the eigenfunctions. The linear growth rates for individual evolution scenarios can be retrieved accurately using the Versatile Advection Code [3], providing an excellent cross-validation of the numerical results. Using VAC, we simply impose the equilibrium configuration and perturb with a cross-stream velocity perturbation. The dominant instability governs the initial evolution so that a growth rate Γ can be derived for specific values for sonic Mach number M, Alfvén number A, and streamwise wavenumber α. The table shows perfect agreement for the growth rates of both subsonic and supersonic sinuous eigenmodes for fixed $A = 0.2$ superAlfvénic cases.

M	A	α	Γ (SPECLS)	Γ (VAC)
0.1	0.2	0.55	0.078228	0.078
1.0	0.2	0.50	0.072307	0.072
2.0	0.2	0.45	0.056635	0.056
3.0	0.2	0.35	0.042155	0.041

Nonlinear evolution

Since the VAC discretization employed is a second-order accurate, shock capturing Total Variation Diminishing scheme (employing Roe's approximate Riemann solver), it is ideally suited for simulating the resulting unsteady, possibly shock-dominated nonlinear regime. Since the eigenfunctions of the most unstable, sinuous mode involve perturbations to far out distances in the cross-stream y-direction when $M \geq 1$, we take the computational domain $[0, -L_y] \times [L_x, L_y]$ according to $L_x = 2\pi/\alpha$ and $L_y = 50$. The figure below shows the density evolution at consecutive times prior to and at about the time $t \simeq 150$ where the perturbed kinetic energy reaches its maximum. The parameters consider a superAlfvénic $A = 0.2$, supersonic $M = 2$ case with $\alpha = 0.45$.

Note how the compressive perturbations reach to large distances away from the initial wake of unit width. The rightward, high speed flow field below and above the wake advects the up-down asymmetric density structures. Nonlinear effects become more pronounced, as these far-field variations can be seen to steepen into fast magnetosonic shocks. The cross-stream wiggling of the wake is clearly further amplified as these shocks develop.

The fast magnetosonic shock type is readily indentified by determining: (i) the shock speed s from the Rankine-Hugoniot conditions that must hold across the discontinuity: e.g. from mass continuity we get $s = \frac{(\rho v_x)_R - (\rho v_x)_L}{\rho_R - \rho_L}$; (ii) in the frame moving with the shock speed s, determining the characteristic slow, Alfvén, and fast Mach number transitions. The analysis shows that the upstream to downstream transition goes from superfast, to subfast but superAlfvénic flow conditions.

Following the exponential growth and – for supersonic cases – the formation of shock structures, the perturbed kinetic and magnetic energy levels decay strongly. Physically, this indicates that the streamwise variation in the system is diminishing and a relaminarization takes place.

At top right in the figure below, we show the evolution of the total kinetic energy

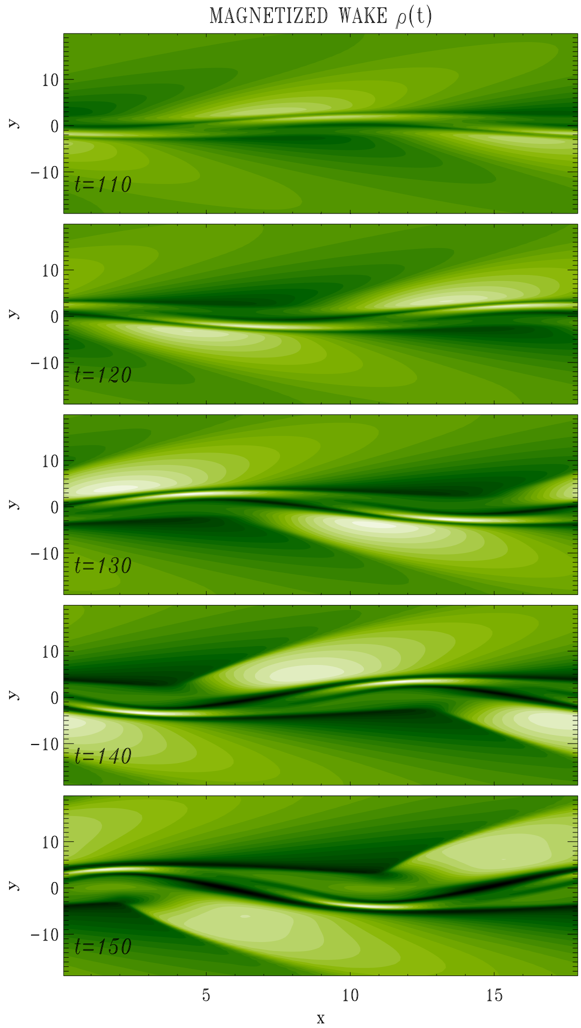
$$E_{kin} = \frac{1}{L_x L_y} \int_{-L_y}^{L_y} \int_0^{L_x} \rho \mid \mathbf{v} \mid^2 \, dx \, dy$$

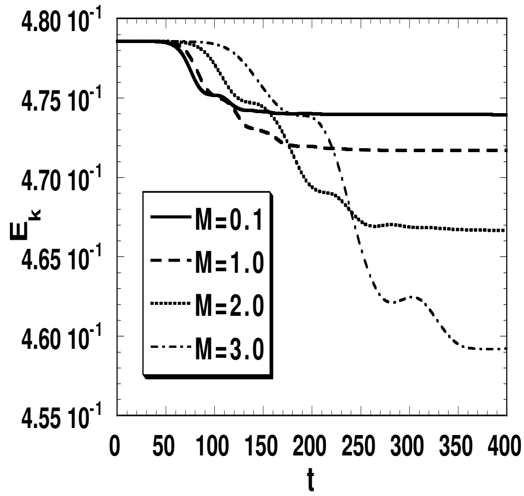
for four cases with $A = 0.2$. When going from subsonic to supersonic regimes, the endstate reaches a lower value for E_{kin}, indicating how kinetic energy has been efficiently converted into thermal and magnetic energy.

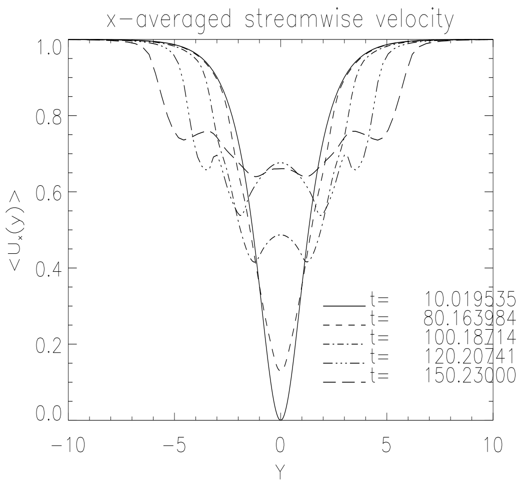
Analyzing the laminar endstate shows that the central part of the wake gets significantly broader and becomes accelerated: for a supersonic case with $M = 2$ and $A = 0.2$, the bottom right panel of the figure shows snapshots of the x-averaged streamwise velocity. The central wake region reaches up to 65 % of the freestream flow. These processes of wake broadening and acceleration are of particular interest for slow solar wind conditions.

Conclusions

We analyzed the linear and nonlinear evolution of flow dominated, magnetized wakes. Both subsonic and supersonic regimes are considered. The most unstable linear mode for these superAlfvénic wakes is of sinuous type (kink-like). Increasing compressibility and magnetic field strength lowers its growth rate.







The nonlinear evolution of a supersonic wake demonstrates how this sinuous mode leads to shock formation. The perturbation reaches far out into the cross-stream direction. Eventually, the wake relaminarizes towards a broadened and centrally accelerated endstate. A forthcoming publication [Dahlburg, Keppens, & Einaudi, *Phys. of Plasmas*, submitted] will address more details.

[1] 'The evolution of a plane jet in a neutral sheet', R.B. Dahlburg, P. Boncinelli, & G. Einaudi, 1998, Phys. of Plasmas **5**, 79.

[2] 'Growth and saturation of the Kelvin-Helmholtz instability with parallel and anti-parallel magnetic fields', R. Keppens, G. Tóth, R.H.J. Westermann, & J.P. Goedbloed, 1999, J. Plasma Phys. **61**, 1.

[3] 'A general code for modeling MHD flows on parallel computers: Versatile Advection Code', G. Tóth, 1996, Astrophys. Lett. and Comm. **34**, 245.