

## Chapter 2

### A Model of Intergenerational Mobility Variation over Time and Place\*

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A rapidly growing literature, surveyed in Solon (1999), is examining the empirical association between the incomes of parents and their children. With the acquisition of new data, researchers recently have begun to explore the ways in which intergenerational income mobility varies between countries and over time. Solon (2002) summarizes the new international evidence, which is substantially expanded by several of the chapters in this book. In addition, Reville (1995), Fertig (2001), and the authors of Chapter 5 have begun to study temporal change in intergenerational mobility in the United States, and the authors of Chapter 6 address that subject for the United Kingdom. This new research on intergenerational mobility variation over time and place is important both because it documents important features and trends in income inequality and because it may produce valuable clues about *how* income status is transmitted across generations.

The purpose of this chapter is to present a theoretical framework for interpreting the evidence from this newly emerging literature. I begin by modifying the Becker-Tomes (1979) model in a way that rationalizes the log-linear intergenerational income regression commonly estimated by empirical researchers. Analysis of the model shows that the steady-state intergenerational income elasticity increases with the heritability of income-related traits, the efficacy of human capital investment, and the earnings return to human capital, and it decreases with the progressivity of public investment in human

capital. Cross-country differences in both intergenerational mobility and cross-sectional income inequality could arise from differences in any of these factors.

The model also can be used to understand intergenerational mobility changes over time. For example, should we expect that the recent era of increasing earnings inequality in the United States and other countries also has been an era of decreasing intergenerational mobility? To address such questions, I use the model to examine how the intergenerational elasticity changes when the steady state is perturbed by an innovation to either the earnings return to human capital or the progressivity of public investment in human capital. The results suggest that an increase in the earnings return to human capital tends to decrease intergenerational mobility, and a shift to more progressive public investment in human capital tends to increase it.

## 1. Assumptions

Assume for simplicity that family  $i$  contains one parent of generation  $t-1$  and one child of generation  $t$ . The family must allocate the parent's lifetime after-tax earnings  $(1 - \tau)y_{i,t-1}$  between the parent's own consumption  $C_{i,t-1}$  and investment  $I_{i,t-1}$  in the child's human capital. The resulting budget constraint,

$$(1 - \tau)y_{i,t-1} = C_{i,t-1} + I_{i,t-1}, \quad (1)$$

assumes that the parent cannot borrow against the child's prospective earnings and does not bequeath financial assets to the child. See Becker and Tomes (1986) for an analysis that relaxes this assumption. Given the simplifying assumption of proportional taxation at rate  $\tau$ , redistributive government policy will be represented in this analysis solely by progressive public investment in children's human capital.

The technology translating the investment  $I_{i,t-1}$  into the child's human capital  $h_{it}$  is

$$h_{it} = \theta \log(I_{i,t-1} + G_{i,t-1}) + e_{it} \quad (2)$$

where  $G_{i,t-1}$  is the government's investment in the child's human capital (for example, through public provision of education or health care),  $\theta > 0$  represents a positive marginal product for human capital investment, the semi-log functional form imposes decreasing marginal product, and  $e_{it}$  denotes the human capital endowment the child receives regardless of the investment choices of the family and government. This endowment represents the combined effect of many child attributes influenced by nature, nurture, or both. In the words of Becker and Tomes (1979, p. 1158), children's endowed attributes "are determined by the reputation and 'connections' of their families, the contribution to the ability, race, and other characteristics of children from the genetic constitutions of their families, and the learning, skills, goals, and other 'family commodities' acquired through belonging to a particular family culture. Obviously, endowments depend on many characteristics of parents, grandparents, and other family members and may also be culturally influenced by other families."

With this characterization of the sources of the endowment, it is natural to assume that the child's endowment  $e_{it}$  is positively correlated with the parent's endowment  $e_{i,t-1}$ .

I follow Becker and Tomes (1979) in assuming that  $e_{it}$  follows the first-order autoregressive process

$$e_{it} = \delta + \lambda e_{i,t-1} + v_{it} \quad (3)$$

where  $v_{it}$  is a white-noise error term and the heritability coefficient  $\lambda$  lies between 0 and 1.<sup>1</sup>

Finally, the child's income  $y_{it}$  is determined by the semi-log earnings function

$$\log y_{it} = \mu + ph_{it} \quad (4)$$

where  $p$  is the earnings return to human capital. Following Juhn, Murphy, and Pierce (1993), I will characterize an era of greater earnings inequality as an era of higher  $p$ . This era need not exhibit higher *levels* of earnings because the higher  $p$  might be accompanied by a lower  $\mu$ .

## 2. Family Investment Behavior

Suppose the parent divides her or his after-tax income  $(1 - \tau)y_{i,t-1}$  between own consumption  $C_{i,t-1}$  and investment  $I_{i,t-1}$  in the child's human capital so as to maximize the Cobb-Douglas utility function

$$U_i = (1 - \alpha) \log C_{i,t-1} + \alpha \log y_{it} . \quad (5)$$

The altruism parameter  $\alpha$ , which lies between 0 and 1, measures the parent's taste for  $y_{it}$  relative to  $C_{i,t-1}$ . If the parent is cognizant of equations (1) through (4) and the variables therein, this utility function can be rewritten as

$$U_i = (1 - \alpha) \log[(1 - \tau)y_{i,t-1} - I_{i,t-1}] + \alpha\mu + \alpha\theta p \log(I_{i,t-1} + G_{i,t-1}) + \alpha p e_{it} . \quad (6)$$

Equation (6) expresses the objective function in terms of the choice variable  $I_{i,t-1}$ .

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<sup>1</sup> Although this heritability is partly biological, even the genetic aspect of the process interacts with social behavior in various ways including assortative mating. Lam and Schoeni (1994) and Chadwick and Solon (2002) emphasize the importance of assortative mating for research on intergenerational mobility. The present model, with its single-parent families, sheds no light on the role of assortative mating.

Assuming an interior solution (that is, the level of public investment in the child's human capital is sufficiently low that the parent wishes to augment it with private investment), the first-order condition for maximizing utility is

$$\partial U_i / \partial I_{i,t-1} = -(1-\alpha)/[(1-\tau)y_{i,t-1} - I_{i,t-1}] + \alpha\theta p / (I_{i,t-1} + G_{i,t-1}) = 0. \quad (7)$$

Solving for the optimal choice of  $I_{i,t-1}$  yields

$$I_{i,t-1} = \left[ \frac{\alpha\theta p}{1-\alpha(1-\theta p)} \right] (1-\tau)y_{i,t-1} - \left[ \frac{1-\alpha}{1-\alpha(1-\theta p)} \right] G_{i,t-1}. \quad (8)$$

This remarkably simple result has several intuitive implications. First, holding public investment constant, higher-income parents invest more in their children's human capital. Second, holding taxes constant, higher public investment in a child's human capital partly crowds out the parent's private investment. Third, parents' investment in their children's human capital is increasing in parental altruism  $\alpha$ . Fourth, parental investment also is increasing in  $\theta p$ , which is the earnings return to human capital investment. In other words, parents are more inclined to invest in their children's human capital when the payoff is higher.

### 3. Implications for Steady-State Mobility and Inequality

With these assumptions and results, it is straightforward to derive the implications for the intergenerational income association between  $y_{it}$  and  $y_{i,t-1}$ , and also for the degree of cross-sectional inequality within a generation. Substituting equation (2) into equation (4) yields

$$\log y_{it} = \mu + p[\theta \log(I_{i,t-1} + G_{i,t-1}) + e_{it}]. \quad (9)$$

Then substituting in equation (8) for  $I_{i,t-1}$  and rearranging produce

$$\begin{aligned}
\log y_{it} &= \mu + \theta p \log \left[ \frac{\alpha \theta p}{1 - \alpha(1 - \theta p)} \right] + \theta p \log[(1 - \tau)y_{i,t-1} + G_{i,t-1}] + pe_{it} \\
&= \mu + \theta p \log \left[ \frac{\alpha \theta p(1 - \tau)}{1 - \alpha(1 - \theta p)} \right] + \theta p \log \left\{ y_{i,t-1} \left[ 1 + \frac{G_{i,t-1}}{(1 - \tau)y_{i,t-1}} \right] \right\} + pe_{it}.
\end{aligned} \tag{10}$$

If the ratio  $G_{i,t-1}/[(1 - \tau)y_{i,t-1}]$  is small, this equation can be approximately re-expressed as

$$\log y_{it} \cong \mu + \theta p \log \left[ \frac{\alpha \theta p(1 - \tau)}{1 - \alpha(1 - \theta p)} \right] + \theta p \log y_{i,t-1} + \theta p \{ G_{i,t-1}/[(1 - \tau)y_{i,t-1}] \} + pe_{it}. \tag{11}$$

Equation (11) suggests that intergenerational mobility is influenced by the government's policy for public investment in children's human capital. Suppose that this policy can be characterized as

$$G_{i,t-1}/[(1 - \tau)y_{i,t-1}] \cong \varphi - \gamma \log y_{i,t-1}. \tag{12}$$

A positive value of  $\gamma$  would signify a sort of *relative* progressivity in public investment in children's human capital. With  $\gamma > 0$ , the absolute public investment may or may not be greater for children from high-income families, but the *ratio* of public investment to parental after-tax income decreases with parental income. The more positive  $\gamma$  is, the more progressive is the policy.

Substituting equation (12) into equation (11) yields the regression equation

$$\log y_{it} \cong \mu^* + [(1 - \gamma)\theta p] \log y_{i,t-1} + pe_{it} \tag{13}$$

with intercept  $\mu^* = \mu + \varphi \theta p + \theta p \log \{ \alpha \theta p(1 - \tau)/[1 - \alpha(1 - \theta p)] \}$ . At first glance, equation (13) looks like the log-linear intergenerational income regression frequently estimated by empirical researchers. Viewed as the error term, however,  $pe_{it}$  is not well-behaved. It is

correlated with the regressor  $\log y_{i,t-1}$  because the child's endowment  $e_{it}$  and the parent's log income  $\log y_{i,t-1}$  both depend on the parent's endowment  $e_{i,t-1}$ .

In fact, equation (13) is a familiar entity in introductory econometrics textbooks. It is the first-order autoregression of  $\log y_{it}$  with a serially correlated error term that itself follows a first-order autoregression, as shown in equation (3). In steady state, in which  $\log y_{it}$  and  $\log y_{i,t-1}$  have the same variance, the slope coefficient in the population regression of  $\log y_{it}$  on  $\log y_{i,t-1}$  is equivalent to the correlation between  $\log y_{it}$  and  $\log y_{i,t-1}$ . In the present context, this quantity, which I will denote as  $\beta$ , is the steady-state intergenerational income elasticity. As shown in Greene (2000, pp. 534-535), this quantity is the sum of the two autoregressive parameters, the slope coefficient in equation (13) and the serial correlation coefficient in equation (3), divided by 1 plus their product. Thus, the steady-state intergenerational income elasticity is

$$\beta = \frac{(1-\gamma)\theta p + \lambda}{1 + (1-\gamma)\theta p \lambda}. \quad (14)$$

This is the estimand in most of the empirical literature on intergenerational income mobility.

Equation (14) shows the connection between the commonly estimated intergenerational income elasticity and the structural parameters of this chapter's model. The intergenerational elasticity  $\beta$  is an increasing function of  $\lambda$ ,  $\theta$ ,  $p$ , and  $1-\gamma$ . In other words, the intergenerational elasticity is greater as: (1) the heritability coefficient  $\lambda$  is greater; (2) human capital investment is more productive ( $\theta$  is greater); (3) the earnings return to human capital is greater ( $p$  is greater); and (4) public investment in children's

human capital is less progressive ( $\gamma$  is less positive). The implications for cross-country comparisons are immediate. If country A displays less intergenerational mobility than country B, this could be because country A has stronger heritability, more productive human capital investment, higher returns to human capital, or less progressive public investment in human capital.

The steady-state implications for cross-sectional income inequality also are straightforward to derive. A familiar result from time series analysis is that the first-order autoregression with a first-order autoregressive error term in equation (13) can be rewritten as a second-order autoregression with a “white noise” error term:

$$\log y_{it} = (1 - \lambda)(\mu^* + p\delta) + [(1 - \gamma)\theta p + \lambda]\log y_{i,t-1} - [(1 - \gamma)\theta p \lambda]\log y_{i,t-2} + pv_{it}. \quad (15)$$

Then, the standard result on the variance of a variable following a stationary second-order autoregression<sup>2</sup> can be used to derive the cross-sectional variance of log income within a generation:

$$Var(\log y_{it}) = \frac{[1 + (1 - \gamma)\theta p \lambda]p^2 Var(v_{it})}{[1 - (1 - \gamma)\theta p \lambda](1 - \lambda^2)\{1 - [(1 - \gamma)\theta p]^2\}} \quad (16)$$

where  $Var(v_{it})$  is the variance of the innovation in equation (3), the process for heritability of endowments.

Like the intergenerational elasticity  $\beta$ , this expression for  $Var(\log y_{it})$  is an increasing function of  $\lambda$ ,  $\theta$ ,  $p$ , and  $1 - \gamma$ . Thus, like the intergenerational elasticity, cross-sectional income inequality is greater in the presence of stronger heritability, more productive human capital investment, higher returns to human capital, and less

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<sup>2</sup> See Box, Jenkins, and Reinsel (1994, p. 62). I thank Shinichi Sakata, Matthew Shapiro, and Phil Howrey for pointing me to this result.



progressive public investment in human capital. This connection between intergenerational mobility and cross-sectional inequality accords with Bjorklund and Jantti's (1997) conjecture that the contrasts between Sweden and the United States in both intergenerational mobility and inequality may be related to each other. The mapping between intergenerational mobility and cross-sectional inequality, however, is not exact because the expression for  $Var(\log y_{it})$  in equation (16) also depends on  $Var(v_{it})$ , which does *not* appear in the expression for  $\beta$  in equation (14). Thus, two countries with approximately the same intergenerational elasticity might differ in cross-sectional inequality because they differ in their heterogeneity of endowed income-related traits.

#### 4. Departures from the Steady State

Numerous writers have raised the question of whether the increase in earnings inequality that has occurred since the late 1970's has been accompanied by a decline in intergenerational mobility. While this ultimately is an empirical question, interpretation of the empirical evidence will benefit from a theoretical perspective. It is straightforward to use this chapter's model to examine how the intergenerational elasticity responds to perturbations from the steady state.

Suppose that society is in steady state in generation  $t - 1$ , but earnings inequality increases in generation  $t$ . Following Juhn, Murphy, and Pierce (1993), I represent the increased earnings inequality as an increase from  $p$  to  $p_t$  in the earnings return to human capital. As Chapter 5 points out, however, at the same time that earnings inequality increased in the United States, public investment in human capital arguably became more progressive. Chapter 6 suggests that the progressivity of public investment moved in the

opposite direction in the United Kingdom. I represent a change in the progressivity of public investment as a shift from  $\gamma$  to  $\gamma_t$  that is known by the parents in generation  $t-1$  at the time that they choose how much of their own income to invest in the children of generation  $t$ .

The intergenerational income elasticity between generations  $t$  and  $t-1$  is

$$\beta_t = \text{Cov}(\log y_{it}, \log y_{i,t-1}) / \text{Var}(\log y_{i,t-1}). \quad (17)$$

This no longer is equivalent to the intergenerational correlation because  $\log y_{it}$  and  $\log y_{i,t-1}$  have different variances. Some tedious algebra shows that

$$\beta_t = \frac{p_t}{p} \left[ \frac{(1-\gamma_t)\theta p + \lambda + (\gamma - \gamma_t)(1-\gamma)\theta^2 p^2 \lambda}{1 + (1-\gamma)\theta p \lambda} \right]. \quad (18)$$

Although equation (18) is cumbersome, it yields very straightforward results for two special cases.

First, suppose that earnings inequality increases, but the progressivity of public human capital investment stays constant with  $\gamma_t = \gamma$ . Then equation (18) simplifies to

$$\beta_t = \frac{p_t}{p} \left[ \frac{(1-\gamma)\theta p + \lambda}{1 + (1-\gamma)\theta p \lambda} \right]. \quad (19)$$

This is simply the steady-state elasticity  $\beta$  from equation (14) inflated by the factor  $p_t/p$ . This result provides formal support for the common intuition that, other things equal, an increase in earnings inequality might be expected to result in a higher intergenerational elasticity. It is worth noting that equation (19) holds regardless of whether the parents anticipate the change from  $p$  to  $p_t$ .

Second, suppose that public human capital investment becomes more progressive, but the return to human capital stays constant with  $p_t = p$ . Then  $\beta_t$  equals just the

bracketed expression in equation (18). If one subtracts the steady-state elasticity in equation (14) from this new elasticity, one finds that the change in the intergenerational elasticity is

$$\beta_t - \beta = (\gamma - \gamma_t)\theta p. \quad (20)$$

Thus an increase in the progressivity of public human capital investment leads to a decrease in the intergenerational income elasticity.

## 5. Conclusion

In this chapter, I have developed a simple model in which optimizing behavior by families leads to the log-linear intergenerational income regression equation commonly estimated by empirical researchers. The steady-state intergenerational income elasticity turns out to be a straightforward function of parameters representing four key factors: the strength of the “mechanical” (for example, genetic) transmission of income-generating traits, the efficacy of investment in children’s human capital, the earnings return to human capital, and the progressivity of public investment in children’s human capital. The implication is that, if country A displays less intergenerational mobility (a higher intergenerational income elasticity) than country B, this could be because country A has stronger heritability, more productive human capital investment, higher returns to human capital, or less progressive public investment in human capital. These same factors also tend to increase cross-sectional income inequality. In addition, an analysis of perturbations of the steady state suggests that an era of rising returns to human capital or declining progressivity in public human capital investment is also an era of declining intergenerational mobility.

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