Math 224—Project 3
A little bit of population control...

by Gavin LaRose (glarose@NebrWesleyan.edu), Nebraska Wesleyan University, March 1998

EcoSystems, Inc.
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20 April 1998

Rigorous Mathematical Contractors (RiMaC), Inc.
Suite 3, Strawmarket Business Plaza
Lonlinc, SK 04685

Dear RiMaC:

As you undoubtedly know, EcoSystems (Inc.) is a thriving company founded to bring ecologically sound farm-raised fish to the scenic Leseatt area of Tonwashing in our bright blue environmentally friendly delivery vans. Sensing a burgeoning market in the area of similarly conscientious coffee production, we have purchased a small coffee-growing island in the Pacific. To our dismay, however, this was beset by horrible coffee-plant eating insects, which we dealt with in accordance to the environmentally conscious tenets of our company by introducing to the island of a species that had not been present there, the orange-backed red-tongued horny toad (*Bufo arangus*). As we expected, this voracious predator did eat the insects—but then continued to increase in population, and the island is now overrun with the toads. As it is (1) getting messy to drive about, and (2) getting expensive to do the requisite cleaning of our bright blue environmentally friendly coffee bean moving trucks, we are consulting with your company on a possible solution to this Toad Problem.

We have been informed by a local biologist that this toad is susceptible to a viral infection unique to that species known as *frigus-communis* that kills infected toads after some short period of infection. We are therefore considering releasing some number of infected toads to the island, in hopes that this will reduce the overal number of toads to some reasonable level, and are contacting you to investigate the possible results of this procedure mathematically. The number of toads currently on the island is approximately at the island's carrying capacity, and we expect from preliminary discussions with a peculiarly humorous mathematician in your area (the notorious Dr. the-P-is-very-important Gavin LaRose) that resulting changes in toad population would be modeled approximately by the system

$$N' = (a-b)N(1-N/L) - dI$$
$$I' = rI(N-I) - (b+d)I.$$

In this, N is the total toad population, I the population of infected toads, a and b the natural toad birth and death rates, d the infection induced death rate, and r the infection rate. While these parameters are at best difficult to measure, we expect that the ratio of the infection death rate to the difference between a and b should be somewhat less than one, as should the ratio of b to this difference. Finally, the product rL should be approximately 10 times the difference $a-b$.

Based on this information, we need from you a complete investigation of what will happen to the toad population on the island following the introduction of some moderate number of infected toads. Specifically, we would like to know (1) whether this will prove to be an effective way of dealing with the current problem, (2) if so, if the number of infected toads initially released is significant in determining the success of the venture, and (3) (as possible) whether variations in the parameter value estimates above will have an impact on the results you obtain.

As we are getting tired of cleaning the trucks on the island, let alone shaking toads out of our boots in the morning, we are operating under a fairly tight deadline for this project—we would like your final report by the **11th** of **May**. To facilitate this, we have arranged with our great friend and patron, the aforementioned

Dr. LaRose, to answer any questions you might have in the process of working on this problem. We look forward to hearing from you.

Sincerely,
"Chuck" R. D. Arwin
President, EcoSystems, Inc.

crda:glr