Math 224—Project 2
Buckles, Beams, and such things...

by Gavin LaRose (glarose@NebrWesleyan.edu), Nebraska Wesleyan University, March 1998

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9 March 1998

Rigorous Mathematical Contractors (RiMaC), Inc.
Suite 3, Strawmarket Business Plaza
Lonlinc, SK 04685

Dear RiMaC:

As you know, we have here at the Rocket Tech Division of Utoff A.F. Base been involved in the development of the space station Omfreed, a project for which we have utilized your company's formidable mathematical expertise in the past. We are currently considering the structural integrity of the beams that are used to support different sections of the structure, in an effort to determine the manner in which they will respond to the rigors of extended use on the space station.

The situation which we are considering is shown in figure 1. As seen there, there is a central mass m (the ball at the center of the figure) connected to two beams, at the ends of which a force T pressing inwards is imposed. This results in a force F_c pushing the mass upwards. The upwards force is opposed by a structural force in the system, shown in the diagram as F_r. The effect of the force is to produce a displacement y, which is the deflection of the mass from an initially horizontal position.

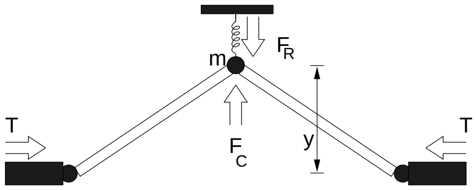

Figure 1: Beam structure showing forces

The engineers that have been working on this problem have suggested that F_r may be modeled as a nonlinear spring force, $F_r = a(y-y_0)^2 + b(y-y_0)^3 + c(y-y_0)$, where y_0 is the initial position of the mass (which may be a measure of initial imperfections in the system) and a, b, and c are constants characterizing the "spring." The compressional force F_c is given approximately by $F_c = 2dy$, where d is again a constant—in this case, measuring the magnitude of the forces T.

Based on this information, we need from you a mathematical model for the deflection of the mass in figure 1, and based on this a prediction for what will happen to the mass as the compressional force F_c is increased. From the work of a resident mathematician who was able to briefly turn her attention to the project, we expect that you should be able to write this model in the form

$$\zeta'' + \gamma\zeta' + \zeta^3 + 2\alpha\zeta^2 + (1-\lambda)\zeta - \lambda\delta = 0,$$

where $\zeta(\tau)$ is a scaled variable measuring the deflection of the mass from the initial position y_0, γ a parameter measuring the damping internal to the system, α a parameter measuring the ratio of different components of the restoring force, δ a parameter measuring the initial position of the mass or system imperfections, and λ a parameter measuring the applied compressional force.

As our manufacturing specifications are at worst exemplary, you should consider in the greatest depth the case $y_0 = 0$, and then give some indication of how the results you obtain change when $y_0 \neq 0$. We would like your results to be as generally stated as possible, but observe that you may find it expedient in some cases to consider $\alpha = 0.75$ and, where appropriate, $\delta = 0.1$.

We are under some pressure to forge ahead with the development of Omfreed, having been beset by delays in other parts of the program. We therefore need your final 4–10 page report by the 6th of April. If you should have questions regarding your investigation, please feel free to contact Dr. Gavin LaRose, a technical expert in many fields, with whom we have an affiliation for this project through the space agency. Please note that you should in any event contact him with an update on your progress on two occasions—on or before the 13th and 30th of March. Sample report formats are also available from Dr. LaRose.

Sincerely,
Lieutenant General Rick N. Backer
Commander, Rocket Tech Division

rnb:glr