Math 224—Project 1
A bit of Decomposing...

by Gavin LaRose (glarose@NebrWesleyan.edu), Nebraska Wesleyan University, January 1998

©1998 Gavin LaRose

Notes on parameters, values & scales:

p	amount of pollution	
x	oxygen concentration	
k	reaction rate for pollution decomposition	
k_1	rate constant for oxygen absorption into lake	
x_m	maximum oxygen concentration in lake	250μM
scale for p	final p value	σ/k
scale for t	pollution time scale	$1/k$
k_1/k	ratio of oxygen absorption to pollution decomposition rates	0.5–5
$\sigma/x_m k$	dimensionless group	?

Lonlinc CPE
the Lonlinc Building
South 9th Avenue
Lonlinc, SK 08685

2 February 1998

Rigorous Mathematical Contractors (R$^{\text{i}}$MaC), Inc.
Suite 3, Strawmarket Business Plaza
Lonlinc, SK 04685

Dear R$^{\text{i}}$MaC:

The Lonlinc CPE (Council on the Protection of the Environment) has recently been put to the task of determining the effect of a number of corporate farms established in the vicinity of Lonlinc. One of these farms is located near the large Eshom Lake, located on the outskirts of Lonlinc, and we are concerned with the possibility of pollution from the farm draining into the lake.

The specific issue we have contacted you to consider is that of the effect of organic pollutants being released into the lake. This release will occur at a rate $\sigma(t)$, after the start of which bacteria in the lake will decompose the pollutants at a rate proportional to the amount of pollutant present. The natural decomposition of pollutants in this manner is desirable in most senses, but also uses up the available oxygen in the lake at the same rate as that at which pollutants are decomposed. The oxygen is replaced by diffusion from the air at the surface of the lake, resulting in the oxygen increasing at a rate proportional to the difference between the maximum oxygen level in the lake, x_m, and the current oxygen level.

We need you to develop a model for the amount of pollutant in the lake as a function of time, and a corresponding model for the oxygen level in the lake. You should consider two possible release rates $\sigma(t)$ in your models: a constant release, and one with a sinusoidal component that has a magnitude approximately one-tenth that of the average release rate. It appears from the preceding discussion that the amount of pollutant in the lake should be independent of the oxygen content there, but that the model for the oxygen content will depend on the amount of pollution in the lake. Based on your models, we need to know what the possible effect of the pollution will be on the oxygen levels in the lake, and under what circumstances we should be concerned for the existing ecosystems in the lake.

Until we have resolved these issues the farm in question is being put off from obtaining a license to operate in the Lonlinc vicinity, a circumstance that puts us in the unenviable position of having to contend with the local politicians who tout economic gains over unproven environmental consequences. We anticipate under these circumstances that large delays may put in jeopardy our funding and therefore need your final 3–8 page report by the 16th of February. We have also been in contact with the estimable if eccentric Dr. Gavin LaRose, who has agreed to act as our liason with you. You should see him with any questions you might have about your work on the project, and should also contact him by the 6th and the 13th of February with an update on your progress. Sample report formats are also available from Dr. LaRose.

We look forward to hearing from you.

Sincerely,

Jack C. Ousteaux
Director, Lonlinc CPE

jco:glr