Math 105—Calculus I Project 3

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Your project solution for the Rocket Tech Division housed at Utoff A.F. Base was so successful that IMC, Inc. has received another contract from Utoff, this one from the Satellite Allocation Division (SAD) of the Base. In light of your work on the previous contract, you have been assigned this one—as the pièce de résistance of your mathematical consulting career at IMC, as it were. The background information from Utoff follows.

Satellite Allocation Division

Utoff A.F. Base
1 Piecemeal Dr.
Haoma, SK
13681-0050

21 November, 1996

Independent Mathematical Contractors, Inc.
Suite 2, Strawmarket Business Plaza
Lonlinc, SK 04685

Dear IMC:

As the nation moves steadily towards the establishment of a space station, there is a necessity to more completely explore and chart the nature of the area of space closest to the Sun. To this end, we are investigating the possibility of putting a near-space satellite (tentatively christened Sidney) in an elliptical orbit about the Sun that would extend in its longest dimension somewhat past the orbit of Mars and in its shortest slightly outside that of Mercury.

A significant consideration for such a satellite is of course the energy, $E(t)$, required to transmit the signal which it returns to Earth. Aspects of this are proportional to the distance from the satellite to Earth. Sidney's elliptical orbit will be given, relative to the Earth's position, by $(x_s(t)/a)^2 - (x_s(t)/a) + (y_s(t)/b)^2 = 1$. Here $x_s(t)$ and $y_s(t)$ are the x and y positions of the satellite as functions of time, and a and b are the major- and minor-axes of the ellipse. A special case which you may find useful to consider is that in which the major- and minor-axes of the elliptical orbit are given to be $a = 7.5 \times 10^{10}$ meters and $b = 2.5 \times 10^{11}$ meters, but we do not require that this special case be explicitly considered in your project report. We need from you some sense of what the maximum and minimum values of $E(t)$ are and, more importantly, where they occur.

In order to submit our analysis of the satellite Sidney to the National Space Agency by the close of the year, we must have your report by the 13th of December. Owing to our close association with the Rocket Tech Division of Utoff, we have commissioned the services of Dr. Gavin LaRose, (with whom we understand you are already somewhat aquainted), whom you may contact with any questions you may have in the process of completing this project. You should also contact him by the 26th of November with an indication of your initial thoughts on the project and by the 6th of December with some evidence of your work towards its completion.

We look forward to receiving your report, which should be typewritten and 3–5 pages in length. Equations explaining your solution should be included in the report, and it is permissible to hand-write these in blank lines between your typewritten explanation if this greatly facilitates the production of the report.

Sincerely,

Dr. Vern R. Von Brawn
Head Scientist, SAD