Institution of Electrical Engineers, 1998. As its title suggests, this book gives more information about programming than about applications.

Finally, the Wikipedia entry for PLC gives some basic information. You can also check out plcopen.org, a vendor-neutral organization promoting standards and interoperability.

Newton's Frames

Q. What is an inertial reference frame? What is inertial space?

Dennis: Those are really good questions, and very fitting for this special issue on inertially stabilized platforms. The concept of an inertial reference frame is one of the most basic in dynamics, and your question deserves an answer.

First, let me explain what a *reference frame* is. A reference frame (not necessarily inertial) consists of three mutually perpendicular directions. The primary function of a reference frame is to keep track of rotations, which are necessarily relative. To keep track of (translational) displacements, which are also necessarily relative, one can choose any point of interest as a reference point. It is customary—although not essential—to associate a reference point with a reference frame, and to think of the reference point as the origin of the reference frame. In fact, a reference frame is often attached to a particle (a point with mass) or a point in a body (a rigid collection of particles having positive size) in order to keep track of displacements as well as rotations. The point of attachment thus serves as the origin of the reference frame.

A reference frame attached to a particle may rotate as the particle moves in space, in which case the reference frame is *position dependent* (spherical and cylindrical frames are examples), while a reference frame attached to a body may rotate as the body rotates, in which case the reference frame is *body fixed*.

Technically speaking, although a reference frame can rotate, it cannot translate. A reference frame cannot translate since its defining vectors have direction and magnitude but no location in space. When we say that a reference frame translates, we mean that its origin translates.

An essential use of reference frames is to keep track of changes in vectors, that is, to determine derivatives. In fact, derivatives of vectors make sense only when the reference frame is specified. When you watch something move, you are, in effect, the reference frame, while the change in the vector from your point of view constitutes a *frame derivative*.

Newton's second law $f = ma$ is valid when the *acceleration vector* a is determined with respect to an inertial reference frame. Consequently, all of dynamics is based on this concept. In defining what an inertial reference frame is, it is common for textbooks to use words such as "absolute" and "stationary." However, nothing in the universe is stationary, and there is no absolute or fixed reference point. The stars move surprisingly quickly with respect to the Earth and the Sun, and likewise everything is moving with respect to everything else. The whole universe could be rotating, but what would that mean? In short, all displacements and all velocities—both translational and rotational—are relative.

So, without being able to say that anything is stationary or absolute, where does that leave us? Let's take the concept of *force* as given; I will not attempt to define it. Now suppose that I have a particle that is *not* subject to any forces, that is, an *unforced particle*. In fact, let's imagine that we have a pair of unforced particles, which could be moving relative to each other. A *relative position vector* of a pair of particles is a position vector with its tail at one particle and its tip at the other particle; there are obviously two such vectors, but this ambiguity is irrelevant. Here is *Newton's first law* (NFL):

> **NFL:** There is a reference frame—called an *inertial reference frame*—with the following property: For every pair of unforced particles, the frame derivative of the particles' relative position vector is constant.

An inertial reference frame is not unique since different inertial reference frames can point in different directions. However, it is easy to show that no inertial reference frame can rotate with respect to any other inertial reference frame.

Newton's first law says that when no forces are acting on the particles, there is a reference frame with respect to which the relative velocity of the particles (that is, the frame derivative of the relative displacement vector) is constant. The origin of the reference frame can be taken to be either of the particles, but that is not essential since the origin of the reference frame is irrelevant to the frame derivative of the relative displacement vector. The phrase "inertial space" is jargon for the directions defined by an inertial reference frame.

The next step is to revisit the pair of particles but now assume that exactly one of the particles is subject to a force f, and consider the relative position vector with tip at the forced particle and tail at the unforced particle. Because of the force, the relative velocity vector with respect to an inertial frame is not constant. How the relative velocity vector changes is given by *Newton's second law* (NSL):

> **NSL:** There is a reference frame—called an *inertial reference frame*—with the following property: For every pair of particles, one of which is unforced, while the other is subject to a force vector f, there exists a positive constant c such that the second frame derivative

of the position vector of the forced particle relative to the unforced particle is cf. The inertia m of the forced particle is defined to be $1/c$.

I have stated NSL in such a way that it includes NFL as a special case. In other words, when the force f is zero, the second frame derivative of the position vector of the forced particle relative to the unforced particle is zero, and thus the frame derivative of the particles' relative position vector is constant.

Notice that this statement of NSL makes no reference to anything being stationary, fixed, or absolute, either translationally or rotationally. As in the case of NFL, the origin of the inertial reference frame with respect to which the frame derivative is taken is irrelevant. It is customary, however, to take the origin of the inertial reference frame to be a convenient point for measuring relative displacements, such as the center of mass of a camera. However, this choice is valid only if the reference point is effectively an unforced particle.

Note that NFL and NSL apply only to particles. By viewing a body as a rigid collection of particles, forces applied to a body can accelerate the body in rotation and translation. The resulting law for rotating bodies relates the angular momentum of the body to the applied moment.

So, what is "inertial space"? In 1852, Foucault demonstrated that the Earth is rotating by showing that a very long pendulum changes its direction of oscillation relative to the Earth as time passes. In fact, the Earth is rotating under the pendulum, which, since no significant forces that can change its direction of swing are acting on it, maintains its direction of motion with respect to an inertial reference frame. To a good approximation, the motion can be viewed as fixed with respect to the stars. Consequently, it is customary to view the stars as being held fixed in "inertial space." However, there is no such thing as "inertial space" since space is space, although, according to NFL and NSL, some frames for keeping track of orientations are inertial.

Newton's laws are not provable, rather, they are axioms. These laws are brilliant insights into how the Universe behaves, and they are invaluable tools for modeling and prediction. A deeper understanding of why and when they are valid or applicable is left to the realm of physics.

ACKNOWLEDGMENTS

Thanks to my personal experts Mark Ardema, David Bayard, Carl Knospe, and Panagiotis Tsiotras for helpful suggestions. Carl suggests the book J. V. Jose and E. J. Saletan, *Classical Dynamics: A Contemporary Approach* (Cambridge University Press, 1998) for a physicists' view of the topic. David points out that the international celestial reference frame (ICRF) based on distant galaxies is used by the astronomy community. For details, see http://rorf.usno.navy.mil/ICRF/.