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Wind-field reconstruction from flight data using an unbiased minimum-variance unscented filter

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Although guidance of all aircraft is affected by wind disturbances, micro-unmanned aerial vehicles are especially susceptible. To estimate unknown wind disturbance, we consider two illustrative scenarios for planar flight. In the first scenario, we assume that measurements of the heading angle are available, while, in the second scenario, we assume that measurements of the heading angle are not available. Since the disturbance estimation problem is non-linear, we develop an extension of the unscented Kalman filter that provides an estimate of the unknown wind disturbance. Furthermore, we show through simulations that, when the heading angle is not measured, a kinematic ambiguity is introduced. However, when the initial heading angle is known and the subsequent heading angle is not measured, this kinematic ambiguity is resolved and accurate estimates of the wind velocity are obtained.

Key words: input reconstruction, wind estimation, UAV

Notation

x, y	Ground coordinates of UAV
ψ	Heading angle of UAV
ω	Steering rate

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$V_{AC/W}$	Airspeed of UAV
$V_{W/E}, \varphi$	Magnitude and direction of wind velocity, respectively
$V_{AC/E}, \theta$	Magnitude and direction of aircraft velocity, respectively
$V_{W/E,x}, V_{W/E,y}$	x and y component of wind velocity, respectively
x_k	State vector at time step k
y_k	Output vector at time step k
u_k	Known input vector at time step k
w_k, v_k	Process noise and sensor noise at time step k, respectively
A_k, B_k, C_k, G_k	State-space matrices at time step k
$\hat{x}_0, R_k$	Mean and covariance of the initial state vector, respectively
Q_k, R_k	Process noise covariance and sensor noise covariance at time step k, respectively
$J(x_k)$	Cost function
$\rho(p\backslash q)$	Conditional probability density function of p given q
$\hat{x}_k$	State estimate
n	Order of a system (dimension of the state vector)
$\hat{x}_{k\backslash k-1}$	State estimate after forecast step at time step k
$\hat{y}_{k\backslash k-1}$	Estimate of output after forecast step at time step k
$P^{xx}_{k\backslash k-1}, P^{yy}_{k\backslash k-1}, P^{xy}_{k\backslash k-1}$	State error covariance, output error covariance, and cross error covariance after forecast step at time step k
$P^{xx}_{k\backslash k}$	Error covariance at time step k
K_k	Kalman filter gain at time step k
e_k	Unknown input vector at time step k
H_k	Unknown-input matrix at time step k
Π_k, V_k	Intermediate quantities in the computation of unbiased minimum-variance filter gain
L_k	Unbiased minimum-variance filter gain at time step k
$\hat{e}_k$	Estimate of the unknown input at time step k
f_k, h_k	Functions representing the process and observation models, respectively
x^a_k	Augmented state vector at time step k
n_a	Dimension of the augmented state vector
p^{xxa}_k	Augmented error covariance at time step k
χ_k	Sigma-point matrix at time step k
$\text{col}_i(A)$	ith column of matrix A
$(A)^{1/2}$	Cholesky square root of matrix A
$\lambda, a, \kappa, \beta$	Parameters of unscented Kalman filter
γ_i	Weights
$1_{p \times q}$	p by q matrix of ones

1. Introduction

Small and micro air vehicles are increasingly being used to improve situational awareness by conducting surveillance, patrolling, and convoy protection (Office of then Secretary of Defense, 2005). These vehicles provide imagery reconnaissance

capability out to five to ten miles at the company/platoon/squad level. Due to their small size, these aircraft have limited payload capacity and usually carry fixed cameras (which require accurate pointing, therefore accurate knowledge of heading) and commercial off-the-shelf autopilots (which often have poor heading-measurement accuracy) (Gross *et al.*, 2006).

Although guidance of all aircraft is affected by the atmospheric motion relative to the Earth, that is, wind, micro-unmanned aerial vehicles (micro-UAVs) are especially susceptible. Localized wind-field estimation, especially winds at low velocity, is difficult. Consequently, alternative means must be used to assess the effects of wind. Efforts in this direction include wind estimation (Osborne and Rysdyk, 2005; Rodriguez and Taylor, 2007), and techniques for path planning in wind, for example (McGee *et al.*, 2006; Ceccarelli *et al.*, 2007), which assume constant known wind fields, and (Rysdyk, 2006; Thomasson, 1998), which make use of gimballed cameras.

In the present paper we develop a technique for using available measurements to estimate the local wind-field velocity. To do this, we use state-estimation techniques that have the ability to reconstruct exogenous disturbance signals that are not directly measured.

In the case of linear systems, early work on reconstructing exogenous signals includes input reconstruction through system inversion (Sain and Massey, 1969; Moylan, 1977), while methods using optimal filters are developed in (Glover, 1969; Corless and Tu, 1998; Hou and Patton, 1998; Xiong and Saif, 2003). More recently, a technique for reconstructing unknown exogenous disturbances has been developed in (Gillijns and De Moor, 2005; Palanthandalam-Madapusi *et al.*, 2006; Palanthandalam-Madapusi and Benrstein, 2007) as an extension of unbiased minimum-variance filtering (Kitanidis, 1987).

In this paper, we extend the techniques in (Palanthandalam-Madapusi and Benrstein, 2007) for estimating unknown external disturbances for non-linear systems. This technique is based on the unscented Kalman filter (UKF) (Julier *et al.*, 2000; Julier and Uhlmann, 2004) for state estimation for non-linear systems, which is an example of sigma-point Kalman filters (SPKF) (van der Merwe *et al.*, 2004). Recent work (Julier *et al.*, 2000; van der Merwe *et al.*, 2004) illustrates the improved performance of SPKFs compared to the extended Kalman filter (EKF), which is prone to numerical problems such as initialization sensitivity, bias (divergence), and instability for strongly non-linear systems.

The nature of the disturbance estimation (input reconstruction) problem depends on the type of measurements available. In the present paper we consider two illustrative scenarios for planar flight. In the first scenario, we assume that measurements of the heading angle are available. In this case, the estimation problem is linear, and the techniques of (Gillijns and De Moor, 2005; Palanthandalam-Madapusi *et al.*, 2006; Palanthandalam-Madapusi and Benrstein, 2007) are applicable. In the second scenario, we assume that measurements of the heading angle are not available. In this case, the disturbance estimation problem is non-linear, and we

therefore develop an extension of the unscented Kalman filter that provides an estimate of the unknown disturbance.

After describing the basic setting in Section 2, the two scenarios described above are developed in Sections 3 and 4. For each scenario, we consider flight involving straight-line and circular motion in the presence of a wind field that varies as a triangular waveform in both of its components. In the case of unknown heading angle, we show that wind field estimation requires knowledge of the initial heading angle in order to remove a kinematic ambiguity.

2.   Wind-field estimation

Consider the planar flight equations

$$\dot{x} = V_{AC/W} \cos \psi + V_{W/E} \cos \phi, \tag{2.1}$$

$$\dot{y} = V_{AC/W} \sin \psi + V_{W/E} \sin \phi, \tag{2.2}$$

$$\dot{\psi} = \omega, \tag{2.3}$$

where x and y are the ground coordinates of the vehicle, $V_{AC/W}$ is the airspeed of the vehicle, ψ is the heading angle, ω is the steering angle rate, $V_{W/E}$ is the horizontal wind speed, and ϕ is the angle of the direction of the horizontal component of wind as measured from the $\hat{\imath}$ axis. The magnitude and direction of velocity of the vehicle with respect to the Earth are $V_{AC/E} \triangleq \sqrt{\dot{x}^2 + \dot{y}^2}$ and $\theta \triangleq \tan^{-1}(\dot{y}/\dot{x})$, respectively; note that $V_{AC/E} = V_{AC/W} + V_{W/E}$. The relationship between the various components of velocities is illustrated in Figure 1. Throughout this paper, we assume that measurements of x and y are available from GPS, and that measurements of $V_{AC/W}$ are available from an airspeed sensor. For simplicity, we assume that $V_{AC/W}$ is aligned with the heading angle ψ as depicted in Figure 1. We consider the problem of estimating the unknown wind speed $V_{W/E}$ and angle ϕ of the wind.
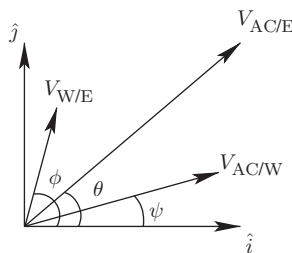
Figure 1 Schematic of relationship between components of velocities in an Earth-fixed frame and the body-fixed frame

3. Measured heading angle

We first consider the case in which the heading angle ψ is measured. In this case, we use (2.1) and (2.2) to estimate $V_{W/E}$ and ϕ. By defining

$$V_{W/E,x} \overset{\triangle}{=} V_{W/E} \cos \phi, \tag{3.1}$$

$$V_{W/E,y} \overset{\triangle}{=} V_{W/E} \sin \phi, \tag{3.2}$$

it follows that (2.1), (2.2) are linear in the unknowns $V_{W/E,x}$ and $V_{W/E,y}$. Once estimates of $V_{W/E,x}$ and $V_{W/E,y}$ are obtained, the wind speed $V_{W/E}$ and angle ϕ can be obtained using the relationships

$$V_{W/E} = \sqrt{V_{W/E,x}^2 + V_{W/E,y}^2}, \tag{3.3}$$

$$\phi = \tan^{-1}\left(\frac{V_{W/E,y}}{V_{W/E,x}}\right). \tag{3.4}$$

Thus the problem is stated as follows.

Problem 1. Equations:

$$\dot{x} = V_{AC/W} \cos \psi + V_{W/E,x}, \tag{3.5}$$

$$\dot{y} = V_{AC/W} \sin \psi + V_{W/E,y}. \tag{3.6}$$

Available measurements: x, y, $V_{AC/W}$, and ψ.
Unknowns: $V_{W/E,x}$ and $V_{W/E,y}$.

Since Problem 1 is linear in the states and linear in the unknowns $V_{W/E,x}$ and $V_{W/E,y}$, we can use the unbiased minimum-variance filter (Palanthandalam-Madapusi and Benrstein, 2007) for linear systems to estimate the states and the unknown inputs. We briefly review the Kalman filter and the unbiased minimum-variance filter.

3.1   Kalman filter

For the linear stochastic discrete-time dynamic system

$$x_k = A_{k-1}x_{k-1} + B_{k-1}u_{k-1} + G_{k-1}w_{k-1}, \tag{3.7}$$

$$y_k = C_k x_k + v_k, \tag{3.8}$$

where $A_{k-1} \in \mathbb{R}^{n \times n}$, $B_{k-1} \in \mathbb{R}^{n \times p}$, $G_{k-1} \in \mathbb{R}^{n \times q}$, and $C_k \in \mathbb{R}^{m \times n}$ are known matrices, the state-estimation problem can be described as follows. Assume that, for all $k \geq 1$,

the known data are the measurements $y_k \in \mathbb{R}^m$, the inputs $u_{k-1} \in \mathbb{R}^p$, and the statistical properties of x_0, w_{k-1} and v_k. The initial state vector $x_0 \in \mathbb{R}^n$ is assumed to be Gaussian with mean $\hat{x}_0$ and error covariance $P_0^{xx} \triangleq \mathrm{E}[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^\mathrm{T}]$. The process noise $w_{k-1} \in \mathbb{R}^q$, which represents unknown input disturbances, and the measurement noise $v_k \in \mathbb{R}^m$, concerning inaccuracies in the measurements, are assumed white, Gaussian, zero mean, and mutually independent with known covariance matrices Q_{k-1} and R_k, respectively. Next, define the cost function

$$J(x_k) \triangleq \rho(x_k|(y_1, \ldots, y_k)), \tag{3.9}$$

which is the conditional probability density function of the state vector $x_k \in \mathbb{R}^n$ given the past and present measured data $y_1, \ldots, y_k$. Under the stated assumptions, the maximization of (3.9) is the state-estimation problem, while the maximizer $\hat{x}_k$ of J is the optimal state estimate.

The optimal state estimate $\hat{x}_k$ is given by the Kalman filter (Kalman, 1960), whose *forecast* step is given by

$$\hat{x}_{k|k-1} = A_{k-1}\hat{x}_{k-1} + B_{k-1}u_{k-1}, \tag{3.10}$$

$$P_{k|k-1}^{xx} = A_{k-1}P_{k-1}^{xx}A_{k-1}^\mathrm{T} + G_{k-1}Q_{k-1}G_{k-1}^\mathrm{T}, \tag{3.11}$$

$$\hat{y}_{k|k-1} = C_k\hat{x}_{k|k-1}, \tag{3.12}$$

$$P_{k|k-1}^{yy} = C_k P_{k|k-1}^{xx} C_k^\mathrm{T} + R_k, \tag{3.13}$$

$$P_{k|k-1}^{xy} = P_{k|k-1}^{xx} C_k^\mathrm{T}, \tag{3.14}$$

where $P_{k|k-1}^{xx} \triangleq \mathrm{E}[(x_k - \hat{x}_{k|k-1})(x_k - \hat{x}_{k|k-1})^\mathrm{T}]$, $P_{k|k-1}^{yy} \triangleq \mathrm{E}[(y_k - \hat{y}_{k|k-1})(y_k - \hat{y}_{k|k-1})^\mathrm{T}]$, and $P_{k|k-1}^{xy} \triangleq \mathrm{E}[(x_k - \hat{x}_{k|k-1})(y_k - \hat{y}_{k|k-1})^\mathrm{T}]$, and whose *data-assimilation* step is given by

$$K_k = P_{k|k-1}^{xy}(P_{k|k-1}^{yy})^{-1}, \tag{3.15}$$

$$\hat{x}_k = \hat{x}_{k|k-1} + K_k(y_k - \hat{y}_{k|k-1}), \tag{3.16}$$

$$P_k^{xx} = P_{k|k-1}^{xx} - K_k P_{k|k-1}^{yy} K_k^\mathrm{T}, \tag{3.17}$$

where $P_k^{xx} \triangleq \mathrm{E}[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^\mathrm{T}]$ is the error-covariance matrix and K_k is the Kalman gain matrix. The notation $\hat{z}_{k|k-1}$ indicates an estimate of z_k at time k based on information available up to and including time $k-1$. Likewise, $\hat{z}_k$ indicates an estimate of z at time k using information available up to and including time k. Model information is used during the forecast step, while measurement data are injected into the estimates during the data-assimilation step, specifically, (3.16).

3.2   Unbiased minimum-variance filter

Consider the system

$$x_k = A_{k-1}x_{k-1} + B_{k-1}u_{k-1} + H_{k-1}e_{k-1} + G_{k-1}w_{k-1}, \tag{3.18}$$

$$y_k = C_kx_k + v_k, \tag{3.19}$$

where x_k, y_k, u_{k-1}, A_{k-1}, B_{k-1}, G_{k-1} and C_k are defined as in Section 3.1, while $e_{k-1} \in \mathbb{R}^l$ represents the unknown input and $H_{k-1} \in \mathbb{R}^{n \times l}$ is the input matrix. We assume that A_{k-1}, B_{k-1}, C_k, D_k, and H_{k-1} are known, while e_{k-1} is unknown.

Owing to the presence of the unknown non-zero-mean term $H_{k-1}e_{k-1}$, the Kalman filter estimate in Section 3.1 is biased in general. The optimal unbiased state estimate $\hat{x}_k$ is given by the Unbiased Minimum-Variance (UMV) filter (Palanthandalam-Madapusi and Benrstein, 2007), whose *forecast* step is given by (3.10)–(3.14), and whose *data-assimilation* step is given by

$$V_k \triangleq C_kH_{k-1}, \tag{3.20}$$

$$\Pi_k \triangleq (V_k^T(P_{k|k-1}^{yy})^{-1}V_k)^{-1}V_k^T(P_{k|k-1}^{yy})^{-1}, \tag{3.21}$$

$$L_k = H_{k-1}\Pi_k + P_{k|k-1}^{xy}(P_{k|k-1}^{yy})^{-1}(I - V_k\Pi_k), \tag{3.22}$$

$$\hat{x}_k = \hat{x}_{k|k-1} + L_k(y_k - \hat{y}_{k|k-1}), \tag{3.23}$$

$$P_k^{xx} = P_{k|k-1}^{xx} - L_kP_{k|k-1}^{yy}L_k^T, \tag{3.24}$$

where L_k is the UMV filter gain matrix. Finally, the estimate of the unknown signal e_{k-1} is given by

$$\hat{e}_{k-1} = H_{k-1}^{\dagger}L_k(y_k - C_k\hat{x}_{k|k-1} - D_ku_k). \tag{3.25}$$

3.3   Results: wind estimation with measured heading angle

The steering angle is chosen to be alternating 38-second periods of zeros and ones, which represents the aircraft flying alternately in a straight line and in circles. The wind-velocity component profiles are chosen to be triangular waveforms. Figure 2 shows the flight path in the absence of wind disturbance, while Figure 3 shows the flight path in the presence of the wind disturbance.

Since Problem 1 is linear in the unknown wind-velocity components, we apply the UMV filter (3.20)–(3.24) and (3.25) to estimate the states and unknown inputs,
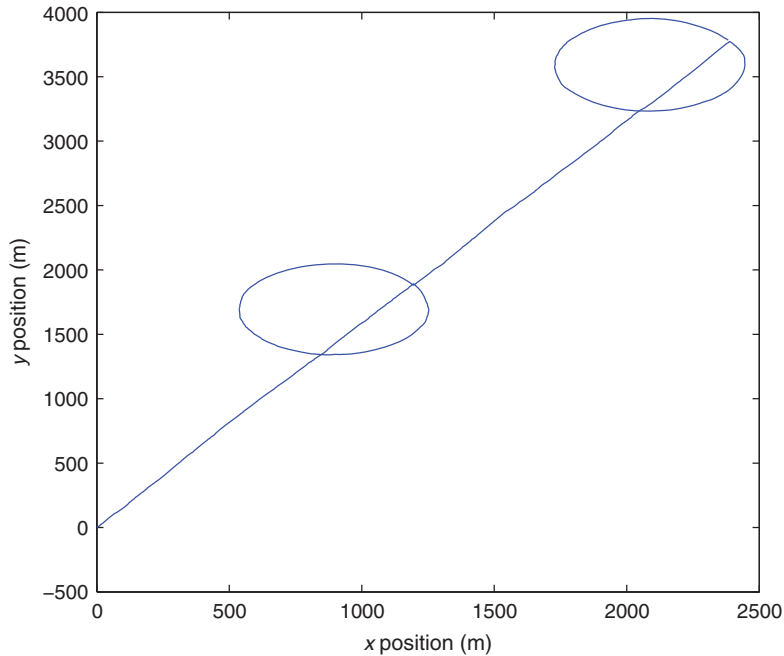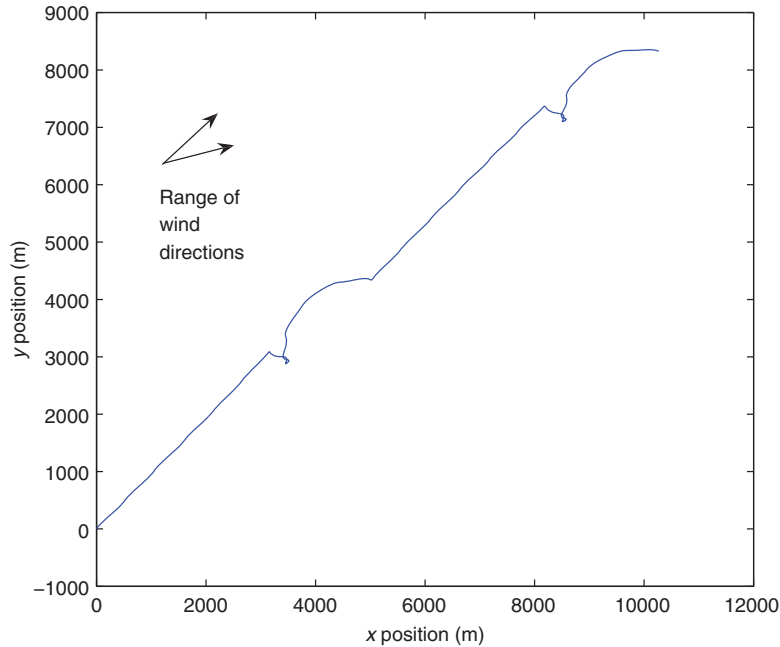
Figure 2 Flight path of the aircraft in the absence of wind disturbance. The steering angle is an alternating 38-second periods of zeros and ones, which represents the aircraft flying in a straight lines and in circles alternately

respectively. To apply the discretetime UMV filter, we discretize Equations (3.5), (3.6) using standard tools in MATLAB. Figure 4 compares the actual flight path and their estimates using the Kalman filter and the UMV filter. Figure 5 shows a magnified version of the time interval from 32 seconds to 48 seconds of Figure 4. Although measurements of x and y positions are available, the state estimates using the UMV filter are seen to be better than the state estimates using the Kalman filter. Finally, Figure 6 shows the actual wind-velocity components and their estimates from (3.25) for both the UMV filter and the Kalman filter, while Figure 7 shows a zoomed-in portion of the interval between 32 seconds and 48 seconds from Figure 6.

   In practice, although measurements of the heading angle ψ are available, they are often unreliable due to the size and cost restrictions of the sensors on a micro-UAV. Hence, we next consider the case in which the heading angle ψ is unknown.

4. Heading angle not measured

We now assume that measurements of the heading angle ψ are not available. Since ψ must be estimated, we consider the complete equations (2.1)–(2.3). Thus the problem can be stated as follows.

Figure 3 Flight path of the aircraft in the presence of wind disturbance. The steering angle is an alternating 38-second periods of zeros and ones, which represents the aircraft flying in a straight lines and in circles alternately. The two arrows show the extremities of the wind direction, which is a time-varying triangular waveform

Problem 2.

$$\dot{x} = V_{AC/W} \cos \psi + V_{W/E,x}, \tag{4.1}$$

$$\dot{y} = V_{AC/W} \sin \psi + V_{W/E,y}, \tag{4.2}$$

$$\dot{\psi} = \omega. \tag{4.3}$$
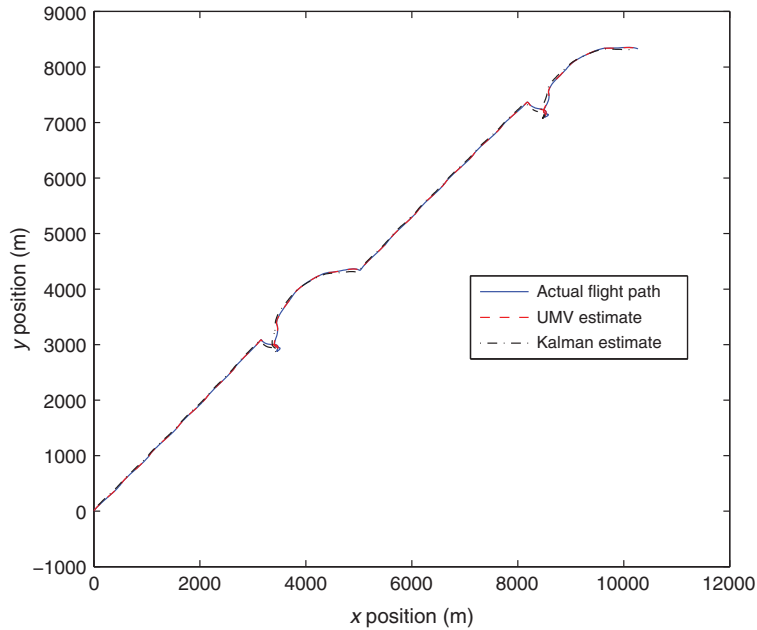
Available measurements: x, y, $V_{AC/W}$, and ω.
Unknowns: ψ, $V_{W/E,x}$, and $V_{W/E,y}$.

In this case, since ψ is not measured the state equations are non-linear. We thus require a filter for non-linear systems.

4.1 State estimation for non-linear systems

Consider the non-linear stochastic discrete-time dynamic system

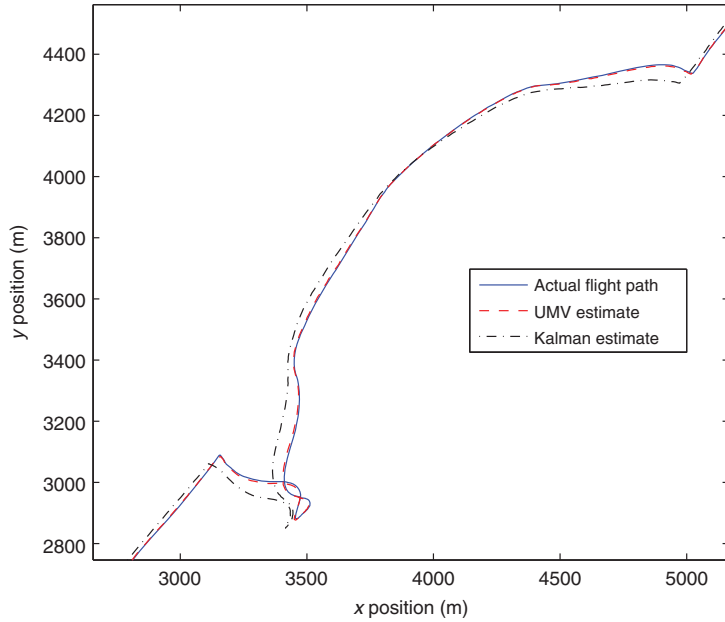$$x_k = f_{k-1}(x_{k-1}, \ u_{k-1}, \ w_{k-1}), \tag{4.4}$$

Figure 4 Actual flight path and estimate of the flight path using the Kalman filter and the unbiased minimum-variance filter in the presence of an unknown wind disturbance

$$y_k = h_k(x_k) + v_k, \tag{4.5}$$

where $f_{k-1} : \mathbb{R}^n \times \mathbb{R}^p \times \mathbb{R}^q \to \mathbb{R}^n$ and $h_k : \mathbb{R}^n \to \mathbb{R}^m$ are, respectively, the process and observation models. The objective of the state-estimation problem is, for all $k \geq 1$, to maximize (3.9). However, the solution to this problem is complicated (Daum, 2005) by the fact that, for non-linear systems, $\rho(x_k|(y_1, \ldots, y_k))$ is not completely characterized by its first and second-order moments. We thus use an approximation based on the classical Kalman filter to provide a suboptimal solution to the non-linear case.

4.2   Unscented Kalman filter

First, for non-linear systems, we consider the unscented Kalman filter (UKF) (Julier and Uhlmann, 2004) to provide a suboptimal solution to the state-estimation problem. Instead of analytically linearizing (4.4)–(4.5) and using (3.10)–(3.17), UKF employs the unscented transform (UT) (Julier *et al.*, 2000), which approximates the posterior mean $\hat{y} \in \mathbb{R}^m$ and covariance $P^{yy} \in \mathbb{R}^{m \times m}$ of a random vector y obtained from the non-linear transformation $y = h(x)$, where x is a prior random vector whose mean $\hat{x} \in \mathbb{R}^n$ and covariance $P^{xx} \in \mathbb{R}^{n \times n}$ are assumed known. UT yields the actual mean $\hat{y}$ and the actual covariance P^{yy} if $h = h_1 + h_2$, where h_1 is linear and h_2 is quadratic (Julier *et al.*, 2000). Otherwise, $\hat{y}_k$ is a *pseudo mean* and P^{yy} is a *pseudo covariance*.

Figure 5 Actual flight path and estimate of the flight path using the Kalman filter and the unbiased minimum-variance filter in the presence of an unknown wind disturbance
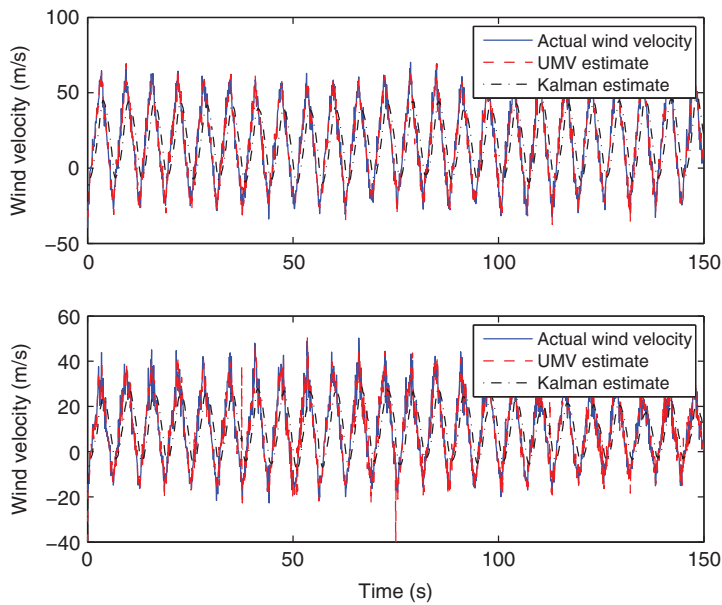
Figure 6 Actual wind velocity and filter estimate when measurements of the heading angle are available
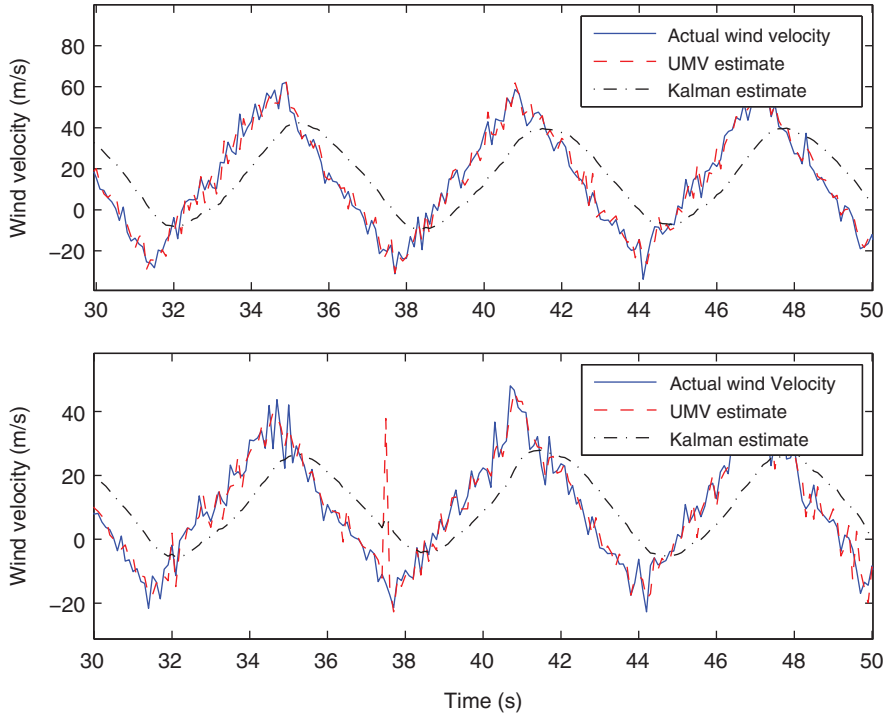
Figure 7 Actual wind velocity and filter estimate when measurements of the heading angle are available

UT is based on a set of deterministically chosen vectors known as sigma points. To capture the mean $\hat{x}_{k-1}^{\mathrm{a}}$ of the augmented prior state vector

$$x_{k-1}^{\mathrm{a}} \triangleq \begin{bmatrix} x_{k-1} \\ w_{k-1} \end{bmatrix}, \tag{4.6}$$

where $x_{k-1}^{\mathrm{a}} \in \mathbb{R}^{n_{\mathrm{a}}}$ and $n_{\mathrm{a}} \triangleq n + q$, as well as the augmented prior error covariance

$$P_{k-1}^{xx\mathrm{a}} \triangleq \begin{bmatrix} P_{k-1|k-2}^{xx} & 0_{n \times q} \\ 0_{q \times n} & Q_{k-1} \end{bmatrix}, \tag{4.7}$$

the sigma-point matrix $\mathcal{X}_{k-1} \in \mathbb{R}^{n_{\mathrm{a}} \times (2n_{\mathrm{a}}+1)}$ is chosen as

$$\begin{cases} \mathrm{col}_0(\mathcal{X}_{k-1}) \triangleq \hat{x}_{k-1}^{\mathrm{a}}, \\ \mathrm{col}_i(\mathcal{X}_{k-1}) \triangleq \hat{x}_{k-1}^{\mathrm{a}} \\ \qquad\qquad + \sqrt{(n_{\mathrm{a}} + \lambda)} \, \mathrm{col}_i\left[\left(P_{k-1}^{xx\mathrm{a}} \right)^{1/2} \right], \quad i = 1, \dots, n_{\mathrm{a}}, \\ \mathrm{col}_{i+n_{\mathrm{a}}}(\mathcal{X}_{k-1}) \triangleq \hat{x}_{k-1}^{\mathrm{a}} \\ \qquad\qquad - \sqrt{(n_{\mathrm{a}} + \lambda)} \, \mathrm{col}_i\left[\left(P_{k-1}^{xx\mathrm{a}} \right)^{1/2} \right], \quad i = 1, \dots, n_{\mathrm{a}}, \end{cases}$$

with weights

$$
\begin{cases}
\gamma_0^{(m)} \triangleq \dfrac{\lambda}{n_a + \lambda}, \\[2ex]
\gamma_0^{(c)} \triangleq \dfrac{\lambda}{n_a + \lambda} + 1 - \alpha^2 + \beta, \\[2ex]
\gamma_i^{(m)} \triangleq \gamma_i^{(c)} \triangleq \gamma_{i+n_a}^{(m)} \triangleq \gamma_{i+n_a}^{(c)} \triangleq \dfrac{1}{2(n_a + \lambda)}, \quad i = 1, \ldots, n_a,
\end{cases}
$$

where $\mathrm{col}_i[(\cdot)^{1/2}]$ is the ith column of the Cholesky square root, $0 < \alpha \leq 1$, $\beta \geq 0$, $\kappa \geq 0$, and $\lambda \triangleq \alpha^2(\kappa + n_a) - n_a$. We set $\alpha = 1$ and $\kappa = 0$ (Haykin, 2001) such that $\lambda = 0$ (Julier and Uhlmann, 2004) and set $\beta = 2$ (Haykin, 2001). Alternative schemes for choosing sigma points are given in (Julier and Uhlmann, 2004).

The UKF *forecast* equations are given by

$$
\mathcal{X}_{k-1} = \left[\hat{x}_{k-1}^a \quad \hat{x}_{k-1}^a 1_{1 \times n_a} + \sqrt{(n_a + \lambda)} \left(P_{k-1}^{xxa} \right)^{1/2} \quad \hat{x}_{k-1}^a 1_{1 \times n_a} - \sqrt{(n_a + \lambda)} \left(P_{k-1}^{xxa} \right)^{1/2} \right],
\tag{4.8}
$$

$$
\mathrm{col}_i(\mathcal{X}_{k|k-1}^x) = f_{k-1}(\mathrm{col}_i(\mathcal{X}_{k-1}^x), \ u_{k-1}, \ \mathrm{col}_i(\mathcal{X}_{k-1}^w)), \quad i = 0, \ldots, 2n_a,
\tag{4.9}
$$

$$
\hat{x}_{k|k-1} = \sum_{i=0}^{2n_a} \gamma_i^{(m)} \mathrm{col}_i(\mathcal{X}_{k|k-1}^x),
\tag{4.10}
$$

$$
P_{k|k-1}^{xx} = \sum_{i=0}^{2n_a} \gamma_i^{(c)} [\mathrm{col}_i(\mathcal{X}_{k|k-1}^x) - \hat{x}_{k|k-1}][\mathrm{col}_i(\mathcal{X}_{k|k-1}^x) - \hat{x}_{k|k-1}]^T,
\tag{4.11}
$$

$$
\mathrm{col}_i(\mathcal{Y}_{k|k-1}) = h_k(\mathrm{col}_i(\mathcal{X}_{k|k-1}^x)), \quad i = 0, \ldots, 2n_a,
\tag{4.12}
$$

$$
\hat{y}_{k|k-1} = \sum_{i=0}^{2n_a} \gamma_i^{(m)} \mathrm{col}_i(\mathcal{Y}_{k|k-1}),
\tag{4.13}
$$

$$
P_{k|k-1}^{yy} = \sum_{i=0}^{2n_a} \gamma_i^{(c)} [\mathrm{col}_i(\mathcal{Y}_{k|k-1}) - \hat{y}_{k|k-1}][\mathrm{col}_i(\mathcal{Y}_{k|k-1}) - \hat{y}_{k|k-1}]^T + R_k,
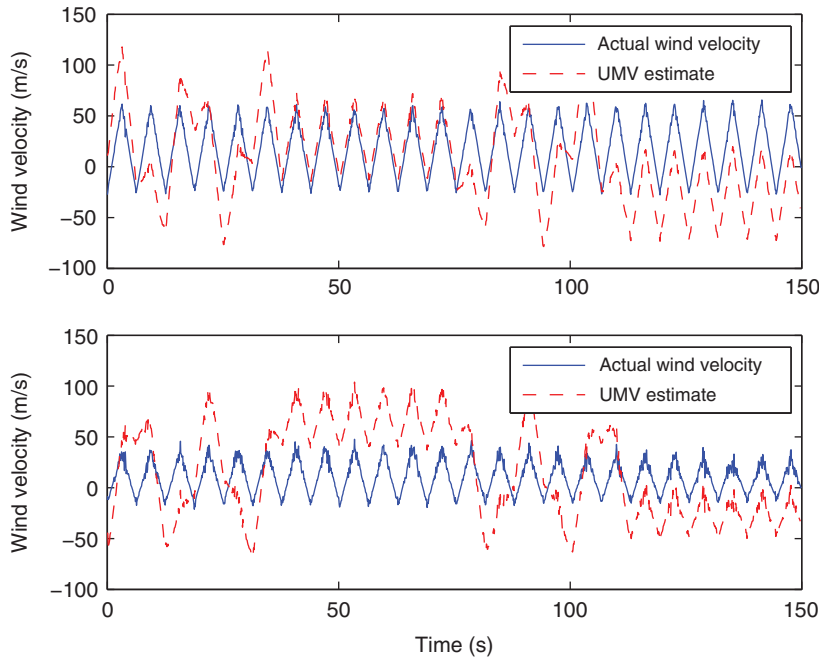\tag{4.14}
$$

$$
P_{k|k-1}^{xy} = \sum_{i=0}^{2n_a} \gamma_i^{(c)} [\mathrm{col}_i(\mathcal{X}_{k|k-1}^x) - \hat{x}_{k|k-1}][\mathrm{col}_i(\mathcal{Y}_{k|k-1}) - \hat{y}_{k|k-1}]^T,
\tag{4.15}
$$

where

$$
\begin{bmatrix} \mathcal{X}_{k-1}^x \\ \mathcal{X}_{k-1}^w \end{bmatrix} \triangleq \mathcal{X}_{k-1}, \mathcal{X}_{k-1}^x \in \mathbb{R}^{n \times (2n_a+1)}, \ \text{and} \ \mathcal{X}_{k-1}^w \in \mathbb{R}^{q \times (2n_a+1)}.
$$

The UKF *data-assimilation* equations are given by (3.15)–(3.17).

Figure 8 Actual wind velocity and filter estimate when the heading angle is not measured. Due to a kinematic ambiguity, accurate estimates of the wind are not obtained

4.3   Unbiased minimum-variance unscented filter

Next, for non-linear systems with unknown inputs, we consider an extension of the UKF along the lines of the linear UMV filter. Thus, to obtain the pseudo mean and the pseudo error covariances we use the unscented transform, and to estimate the states and unknown inputs, we use the expressions derived for the UMV filter. Thus, the *forecast* equations for the unbiased minimum-variance unscented (UMVU) filter are given by (4.8)–(4.15). The *data-assimilation* equations for the UMVU filter are given by (3.20)–(3.24).

4.4   Results: wind estimation with heading angle not measured

To estimate the states and the unknown inputs in Problem 2, we use the UMVU filter described above. We use the same simulation parameters as in the known heading case. Figure 8 shows the actual wind-velocity components and their estimates obtained from the UMVU filter.

As can be seen from Figure 8, the estimates of the wind velocity do not match the actual wind velocity. This is due to the fact that there is a kinematic ambiguity because
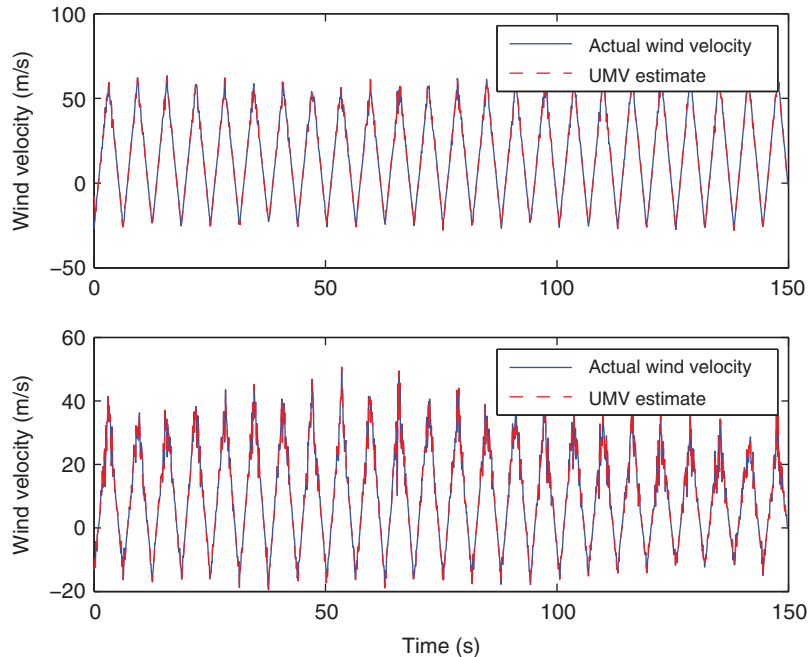
Figure 9 Actual wind velocity and filter estimate when the heading angle is not measured. When the initial heading is assumed to be known, the kinematic ambiguity is resolved and accurate estimates of the wind disturbances are obtained. The initial x and y positions are assumed to be unknown

of the combined effect of unknown heading angle and unknown wind velocity. This kinematic ambiguity is resolved by assuming that the initial heading angle is known. This is a reasonable assumption in practice since many small and micro-UAVs are launched from catapults. However, the initial x and y positions are assumed to be unknown. When the initial heading angle is assumed to be known, but the subsequent heading is not measured, the estimates of the wind-velocity components using the UMVU filter are shown in Figure 9.

5. Conclusions

To estimate unknown wind disturbances, we considered two illustrative scenarios for planar flight. In the first scenario, we assumed that measurements of the heading angle are available. In this case, since the estimation problem is linear, we applied techniques of (Palanthandalam-Madapusi and Benrstein, 2007) to estimate the wind disturbance. In the second scenario, we assumed that measurements of the heading angle were not available. In the second scenario, since the disturbance estimation problem is non-linear, we developed an extension of the unscented Kalman filter that

provided an estimate of the unknown wind disturbance. When the heading angle is not measured, a kinematic ambiguity was introduced. However, when the initial heading angle was known and the subsequent heading angle was not measured, this kinematic ambiguity was resolved and accurate estimates of the wind disturbance were obtained.

References

Ceccarelli, N., *et al.* 2007: Micro AUV Path Planning for Reconnaissance in Wind. *Proceedings of the 2007 American Control Conference,* New York, NY, July 11-13, 6310–16.

Corless, M. and **Tu, J.** 1998: State and input estimation for a class of uncertain systems. *Automatica* 34(6): 757–64.

Daum, F.E. 2005: Nonlinear filters: Beyond the Kalman filter. *IEEE Aerospace and Electronics Systems Magazine* 20(8): 57–69.

Gillijns, S. and **De Moor, B.** 2005: Unbiased minimum-variance input and state estimation for linear discrete-time stochastic systems. *Internal Report ESAT-SISTA/TR 05-228.* Katholieke Universiteit Leuven, Leuven.

Glover, J.D. 1969: The linear estimation of completely unknown systems. *IEEE Transactions on Automatic Control* 14, 766–7.

Gross, D., *et al.* 2006: Cooperative Operations in Urban TERrain (COUNTER). *SPIE Defense and Security Symposium.* Orlando, FL, April.

Haykin S. editor, 2001: *Kalman Filtering and Neural Networks.* Wiley Publishing, New York.

Hou, M. and **Patton, R.J.** 1998: Input observability and input reconstruction. *Automatica* 34(6): 789–94.

Julier, S.J. and **Uhlmann, J.K.** 2004: Unscented filtering and nonlinear estimation. *Proceedings of the IEEE* 92, 401–22.

Julier, S.J., *et al.* 2000: A new method for the nonlinear transformation of means and covariances in filters and estimators. *IEEE Transactions on Automatic Control* 45(3): 477–82.

Kalman, R.E. 1960: A new approach to linear filtering and prediction problems. *Transactions of the ASME – Journal of Basic Engineering* 82, 35–45.

Kitanidis, P.K. 1987: Unbiased Minimum-variance Linear State Estimation. *Automatica* 23(6): 775–78.

McGee, T.G., *et al.* 2006: Optimal Path Planning in a Constant Wind with a Bounded Turning Rate. *AIAA Conference on Guidance, Navigation, and Control*, Keystone, CO.

Moylan, P.J. 1977: Stable inversion of linear systems. *IEEE Transactions on Automatic Control* 22, 74–78.

Office of the Secretary of Defense. 2005: *Unmanned Aircraft Systems Roadmap, 2005–2030.*

Osborne, J. and **Rysdyk, R.** 2005: Waypoint Guidance for Small UAVs in Wind. *AIAA Infotech@Aerospace.* Arlington, VA.

Palanthandalam-Madapusi, H.J. and **Benrstein, D.S.** 2007: Unbiased minimum-variance filtering for input reconstruction. *Proceedings of the 2007 American Control Conference,* New York, NY, July.

Palanthandalam-Madapusi, H., *et al.* 2006: System identification for nonlinear model updating. *Proceedings of the 2006 American Control Conference,* Minneapolis, MN, June.

Rodriguez, A. and **Taylor, C.** 2007: Wind Estimation Using an Optical Flow Sensor on a Miniature Air Vehicle. *AIAA Conference on Guidance, Navigation and Control*, Hilton Head, SC, August.

Rysdyk, R. 2006: Unmanned Aerial Vehicle Path Following for Target Observation in Wind. *Journal of Guidance, Control, and Dynamics* 29(5): 1092–100.

Sain, M.K. and **Massey, J.L.** 1969: Invertibility of linear time-invariant dynamical systems. *IEEE Transactions on Automatic Control* 14(2): 141–9.

Thomasson, P.G. 1998: Guidance of a Roll-Only Camera for Ground Observation in Wind. *Journal of Guidance, Control, and Dynamics* 21(1): 39–44.

van der Merwe, R., *et al.* 2004: Sigma-point Kalman filters for nonlinear estimation and sensor-fusion – applications to integrated navigation. *Proceedings of the AIAA Guidance, Navigation & Control Conference.* Providence, RI.

Xiong, Y. and **Saif, M.** 2003: Unknown disturbance inputs estimation based on a state functional observer design. *Automatica* 39, 1389–98.